LECTURE 7 - DIMENSIONAL ANALYSIS

7.1 MODIVATION

EXAM
- SINGLE SIDED FORMULA SHEET

$\Delta P_\ell = f(D, p, \mu, v)$

$\frac{\Delta P}{\ell}$

5 2 2 5 3

different possible values

↳ 106 combos

Given a problem that involves a high dimensional parameter space:

1. Can we use a limited number of experital data in a laboratory system to inform what will happen in a real system?

2. If yes:

   a. How should we design the lab experiments?

   b. How should we reduce the lab data?

   c. How do we predict real system behavior?

7.2 π-THEOREM (BUCKINGHAM THEOREM)

Background -

1. Physical observations have dimentions (units).
2. All dimentions can be constucted from a small set of "basic dimentions."
   Pressure : $F/A \rightarrow F/L^2$
   Viscosity : $Pa \cdot s \rightarrow F/L^2 \cdot s$
   a. Force, length, and time (FLT)
   b. Mass, length, and time (MLT)
3. All elongations must be homogenious in dimention.
   $(A = B + C)$

π-Theorem:

$$u_1 = f(u_2, u_3, \ldots, u_k)$$

$$\pi_1 = F(\pi_1, \pi_2, \ldots, \pi_{k-r})$$

r = # of reference dimentions, usually 3 (FLT/MLT)

Dimensionless → constucted from $u_1, u_2, \ldots, u_k$

$$\Delta P_\ell = f(D, p, \mu, v) \quad , \quad k = 5, \; r = 3$$

$$\pi_1 = \frac{\Delta P_\ell \, D}{p \, v^2} \quad , \quad \pi_2 = \frac{p \, V D}{\mu}$$

$$\frac{\Delta P_\ell \, D}{p \, V^2} = F\left(\frac{p V D}{\mu}\right)$$

ORGINAL PROBLEM

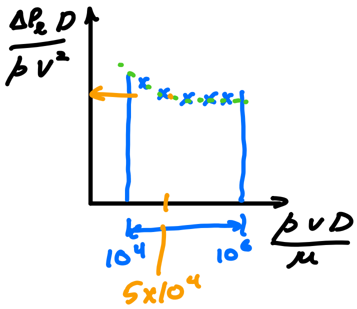
$\left.\begin{matrix} D, u \\ \rho, \mu, V \end{matrix}\right\} \Rightarrow$ range of $\dfrac{\rho V D}{\mu} \rightarrow 10^4 - 10^6$

↳ example

- Design lab expirement so that π_2 is explored in the range identifited.

$D = 6\,in, \rho_{water}, \mu,$ vary "V"

- $\Delta P_\ell \rightarrow$ measure $\pi_1 \rightarrow$ fit a coralation of π_1 as a function of π_2



$$\frac{\Delta P_\ell D}{\rho V^2} = 0.2$$