LECTURE 7 DIMENSIONAL ANALYSIS

7.2 PI THEOREM

$u_1 = f(u_2, \ldots, u_k)$

$\pi_1 = \phi(\pi_2, \ldots, \pi_{k-r})$

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2}, \quad \Pi_2 = \frac{\rho V D}{\mu}$$

7.3 HOW TO CONSTRUCT π GROUPS

1. List all dimentional variables involved in the problem and express them in terms of the basic dimentions (MLT or FLT)
   - Must list all variables, even if they are constant
   - Must understand the problem.

$$\cancel{\Delta p_\ell = f(D, \rho, \mu, V)} \qquad \Delta p_\ell = f(D, \rho, \mu, V, \epsilon)$$

Put these on equ sheet

$$\Delta p_\ell = \frac{\Delta p}{L} \doteq \frac{p}{L} = \frac{FL^{-2}}{L} \doteq FL^{-3} \qquad \mu \doteq \frac{\tau}{\frac{du}{dy}} \doteq \frac{FL^{-2}}{LT^{-1}/L} \doteq FL^{-2}T$$

$$D \doteq L$$

$$\rho \doteq \frac{\gamma}{g} = \frac{FL^{-3}}{LT^{-2}} = FL^{-4}T^2 \qquad \varepsilon = L \qquad V = LT^{-1}$$

surface roughness

2. Determine the number of reference dimension (r) and number of π terms

3. Select a set of repeating parameters

4. Use repeating variables to render other variables dimensionless, thereby building dimensionless groups.

All variables $\Delta p_\ell, D, \rho, \mu, V, \epsilon$ $\quad k=6,\ r=3$

3 repeating variables D, V, ρ

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c \qquad \Pi_2 = \mu D^{a_2} V^{b_2} \rho^{c_2} \qquad \Pi_3 = \epsilon D^{a_3} V^{b_3} \rho^{c_3}$$

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c = FL^{-3}(L)^a (LT)^b (FL^{-4}T^2)^c = F^0 L^0 T^0$$

$$\left.\begin{aligned} F&: 1 + c = 0 \\ L&: -3 + a + b - 4c \\ T&: -b + 2c = 0 \end{aligned}\right\} \rightarrow \begin{aligned} a &= 1 \\ b &= -2 \\ c &= 1 \end{aligned}$$

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2} \rightarrow \text{Dimensionless}$$

$$\Pi_2 = \frac{\mu}{\rho V D} \qquad \Pi_3 = \frac{\epsilon}{D}$$