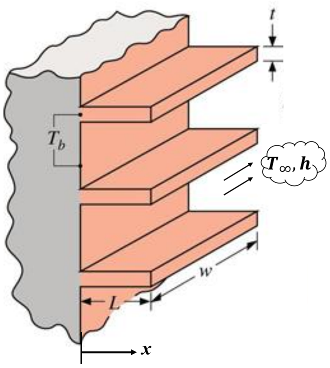
ME3304 Conduction (Previous Test) - Solution

1. Adding extra surface area is common approach to enhance heat transfer. The aluminum fin (k = 240 W/m·K) with uniform cross-sectional area attached to the base surface is one of the typical applications as shown in Figure 1. The temperatures of the base of the fin and surrounding air are T_b = 85 °C and T_∞ = 20 °C, respectively. The convection heat transfer coefficient is h = 50 W/m²·K and radiation is negligible.



Dimensions	Value [unit]
w	*20 mm*
t	*2 mm*
L	*20 mm*

Figure 1 Array of rectangular fins and the key dimension

Assuming the steady-state and temperature is uniform across fin cross section, (a) calculate the temperatures of the fin at x = 5, 15, and 20 mm from the base and (b) plot the profile along x direction. The analytic solutions of the temperature distributions depend on the tip conditions can be found in

Table 1.

Table 1 Temperature distributions and heat rate of fins with uniform cross section

TABLE 3.4 Temperature distribution and heat rates for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
A	Convection: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.75)	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M \tanh mL$	(3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.82)	$M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)
$\theta \equiv T - T_\infty$ $\theta_b = \theta(0) = T_b - T_\infty$	$m^2 \equiv hP/kA_c$ $M \equiv \sqrt{hPkA_c}\theta_b$	A table of hyperbolic functions is given in Appendix B.1.			

Solution:

For the convection tip (Case A from Table 1),

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh ml + (h/mk)\sinh ml}$$

$$where\ m = \sqrt{hP/kA_c} = \sqrt{(50 \times 0.044)/(240 \times 0.00004)} = 15.138\ W$$

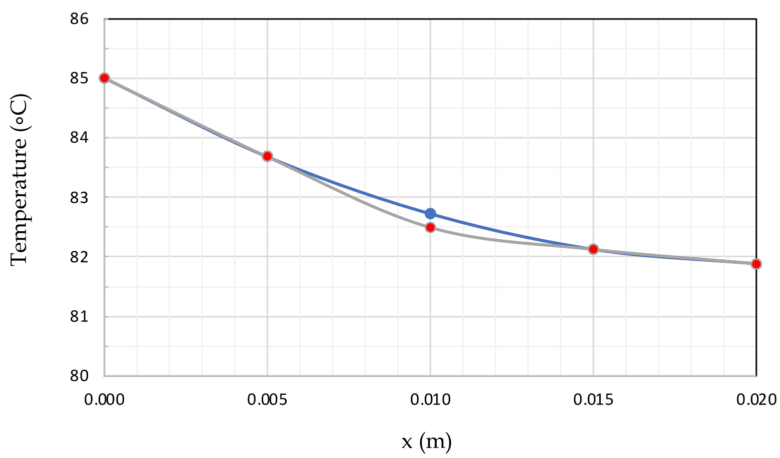
$$T(x) = T_\infty + (T_b - T_\infty)\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh ml + (h/mk)\sinh ml}$$

$$T(x) = 20 + (85-20)\frac{\cosh 15.138(0.02-x) + [50/(15.138 \times 240)]\sinh 15.138(0.02-x)}{\cosh(15.138 \times 0.02) + [50/(15.138 \times 240)]\sinh(15.138 \times 0.02)}$$

Calculate the temperatures along x direction,

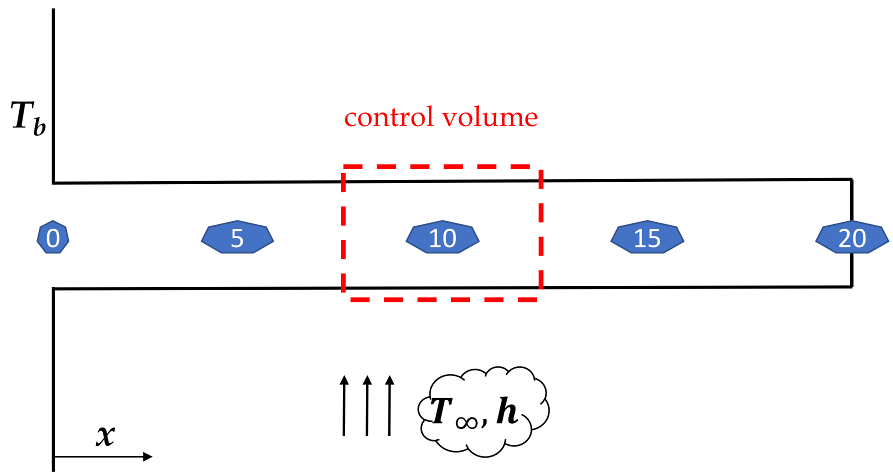
x [mm]	T(°C)	T(K)	Solution Method
5	83.68	356.83	Exact solution
10	82.72	355.87	Exact solution
	82.49	355.65	Finite Difference Analysis
15	82.12	355.27	Exact solution
20	81.88	355.03	Exact solution

Plot of the temperature profile,



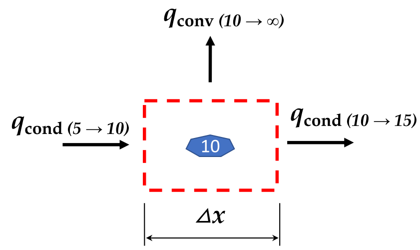
2. From the results of the problem 2 in above, please calculate the temperature of the fin at x = 10 mm applying the finite difference analysis.

a) Perform energy balance around the control volume & write the equation of energy balance
b) Calculate the temperature at x = 10mm and add the result to the plot of temperature profile of problem 2.



Solution:

c) Perform energy balance around the control volume & write the equation of energy balance



$$q_{cond(5\to10)} - q_{cond(10\to15)} = q_{conv(10\to\infty)} = 0$$

$$-kA_c\frac{T_{10}-T_5}{\Delta x} - \left(-kA_c\frac{T_{15}-T_{10}}{\Delta x}\right) - hA_s(T_{10}-T_\infty) = 0$$

where $A_c = w\times t$, and $A_s = P\times\Delta x = [2(w+t)]\times\Delta x$

d) Calculate the temperature at x = 10mm and add the result to the plot of temperature profile of problem 2.

$$\frac{kA_c}{\Delta x}(-T_{10}+T_5+T_{15}-T_{10}) - hA_s(T_{10}-T_\infty) = 0$$

$$\frac{240(0.02\times0.002)}{0.005}(356.38+355.27-2T_{10}) - 50(0.044\times0.005)(T_{10}-293.15) = 0$$

$$1.92(711.65-2T_{10}) - 0.011(T_{10}-293.15) = 0$$

T_{10} = 355.65K or 82.49°C

3. The thin cooper plate is suddenly exposed to the cold gas of 20°C with the uniform heat flux on the upper surface of the plate as shown in the Figure 2. The initial temperature of the copper plate prior to the exposure was 90°C. The insulation plate is attached to the lower surface of the copper plate so that there is no heat loss. The copper plate has the thickness, l of 2mm and the thermal conductivity, k of 400 *W/m·K*. It is assumed that there is no lateral variation of the temperature of the copper plate.

(a) What is the minimum convection heat transfer coefficient to meet the lumped capacitance assumption?
(b) Calculate the temperatures of the copper plate at 1, 2, 4 and 10 seconds by applying the thermal time constant, $\tau_t = 0.5$
(c) Plot the time history of the temperature of the copper plate for t = 0 ~ 10 seconds.

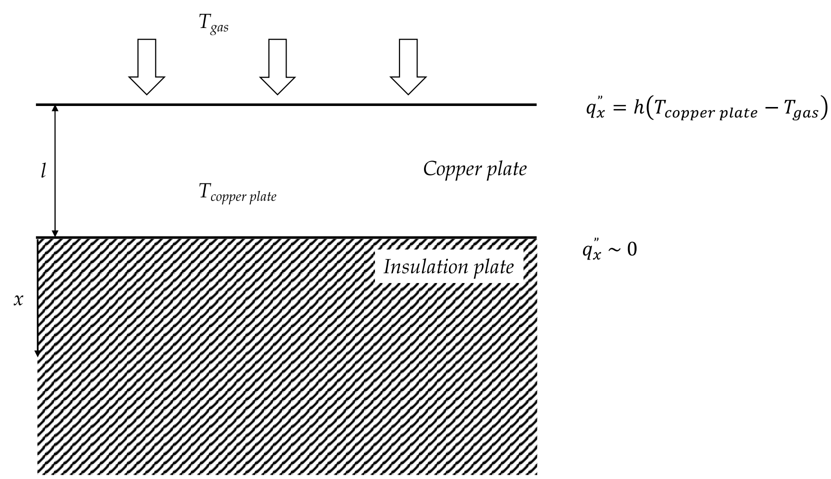


Figure 2 Thermal model of thin copper plate with insulation

Solution:

(a) What is the minimum convection heat transfer coefficient to meet the lumped capacitance assumption?

$$Bi = \frac{h \times l}{k} = \frac{h \times 0.002}{400} < 0.1$$

$$\therefore h < 20{,}000 \; W/m^2K$$

(b) Calculate the temperatures of the copper plate at 1, 2, 4 and 10 seconds by applying the thermal time constant, $\tau_t = 0.5$

Energy balance equation is

$$hA\left[T_{cp}(t) - T_g\right] = \rho V c \frac{dT}{dt}$$

where T_{cp} is the temperature of the copper plate and T_g is gas temperature

$$\frac{T_{cp}(t) - T_g}{T_{cp,i} - T_g} = exp\left(-\frac{hA}{\rho V c}t\right) = exp\left(-\frac{t}{\tau_t}\right)$$

where, $\tau_t = \frac{\rho V c}{hA}$, thermal time constant

$$\therefore T_{cp}(t) = T_g + \left(T_{cp,i} - T_g\right) exp\left(-\frac{t}{\tau_t}\right)$$

For t = 1, 2, 4, and 10 sec, applying the *thermal time constant*, $\tau_t = 0.5$

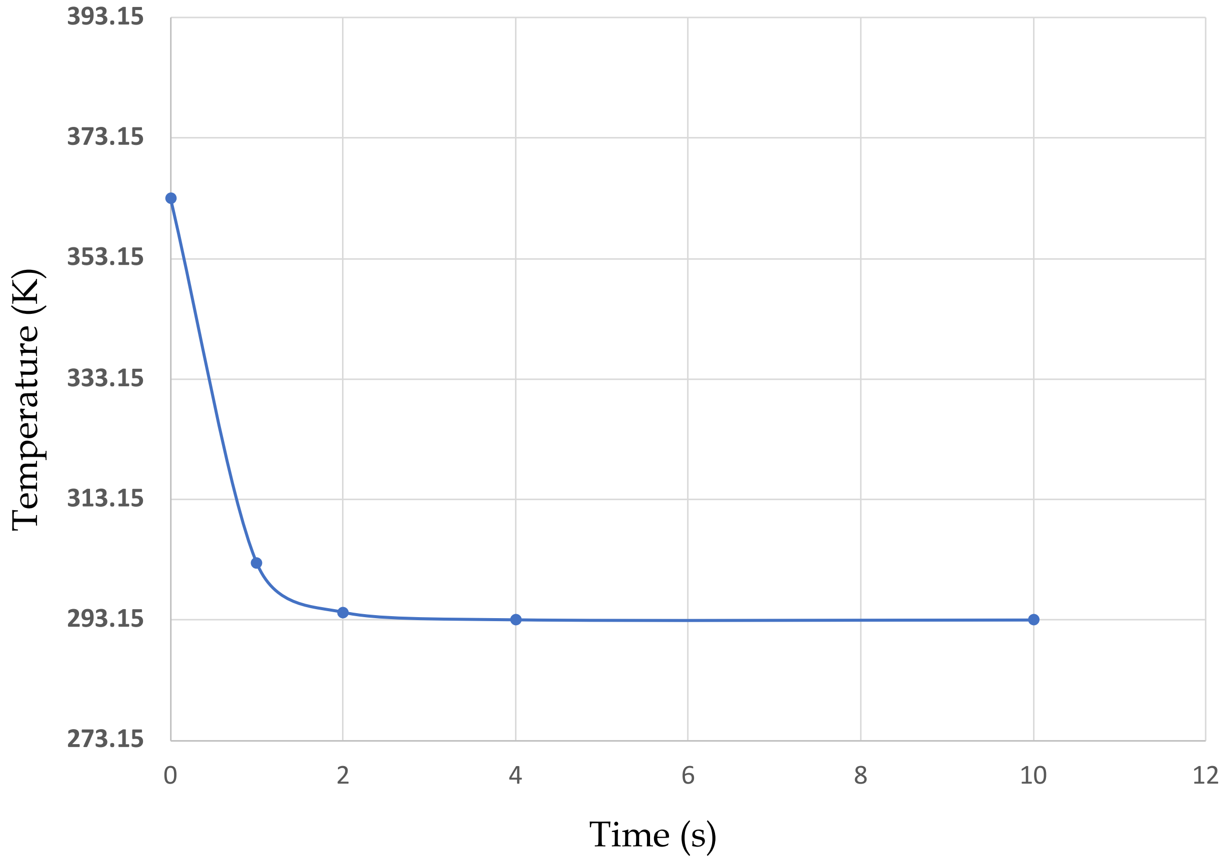
$$\therefore T_{cp}(1) = 293.15 + 70 \times exp\left(-\frac{1}{0.5}\right) = 302.62 \; K$$

$$\therefore T_{cp}(2) = 293.15 + 70 \times exp\left(-\frac{2}{0.5}\right) = 294.43 \; K$$

$$\therefore T_{cp}(4) = 293.15 + 70 \times exp\left(-\frac{4}{0.5}\right) = 293.17 \; K$$

$$\therefore T_{cp}(10) = 293.15 + 70 \times exp\left(-\frac{10}{0.5}\right) = 293.15 \; K$$

(c) Plot the time history of the temperature of the copper plate for t = 0 ~ 10 seconds.



4. A cylinder, 0.1m in length and 0.1m in diameter, is initially at room temperature, 292K. It is exposed in hot gas at 373K with a convection heat transfer coefficient, h of 8500W/m^2·K. Determine the time required for the center of this cylinder to reach 310K. The cylinder has thermal conductivity, k of 1.21W/m·K thermal diffusivity, α of 5.95 x 10^{-7} m^2/s, and were sufficiently long so that it could be considered infinite.

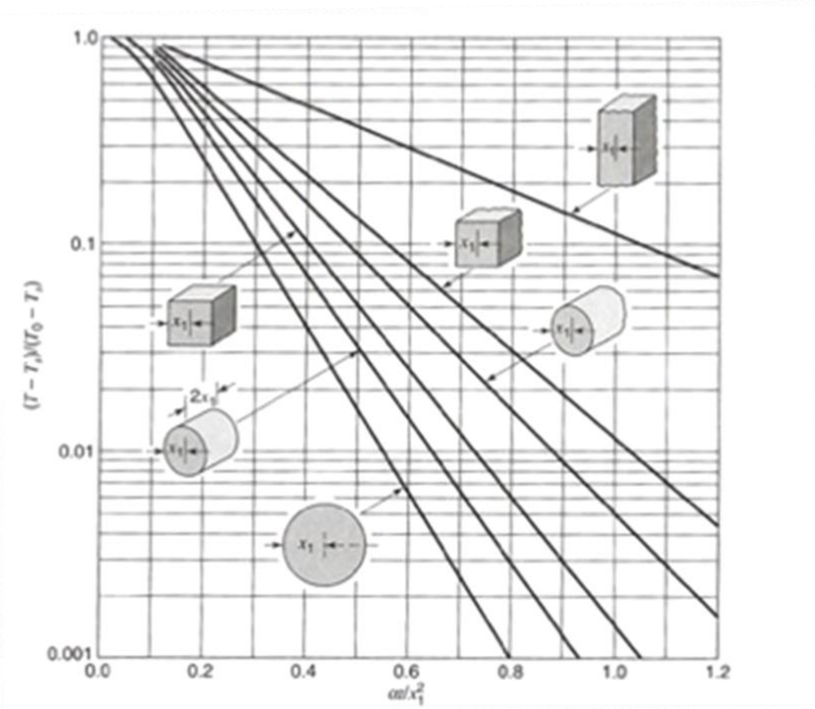


Figure 2 Temperature histories in a semi-infinite solid with surface convection

Solution:

Firstly, evaluate Biot number

$$Bi = \frac{h \times (V/A)}{k} = \frac{h \times (\pi D^2 L/4)/(\pi DL + \pi D^2/2)}{k} = 117$$

For the large value, the data is provided in Figure 2, so that (1) calculated normalized temperature of Y-axis

$$\frac{T - T_s}{T_0 - T_s} = \frac{310 - 373}{292 - 373} = 0.778$$

(2) then, find the corresponding value of x -axis along the line of the cylinder with limited length

$$\frac{\alpha t}{x_1^2} \sim 0.11$$

Therefore,

$$t = 0.11 \times \frac{0.05m^2}{5.95 \times 10^{-7}\, m^2/s} = 462\ s$$

If you chose the infinitely long cylinder, *t* = *546 s*