ME 3304 Heat Transfer

Lecture Note (3) – Convection (Chapter 6 ~ 8)

Chapter 6   Introduction to Convection
Chapter 7   External Flow
Chapter 8   Internal Flow

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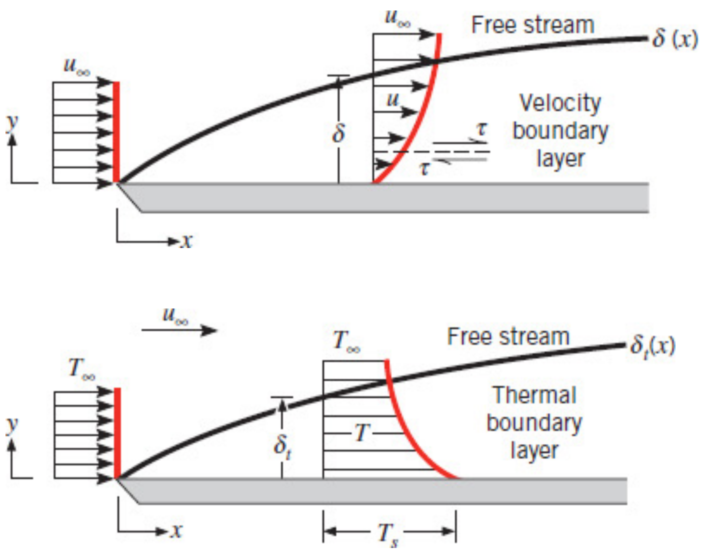
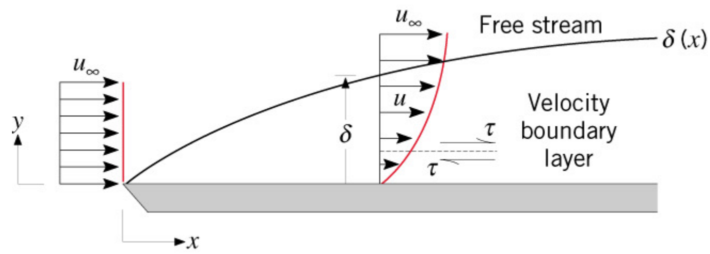
Chapter 6 Introduction to Convection

Velocity & Thermal boundary layer development on an isothermal flat plate



Boundary Layers: Physical Features

- Velocity Boundary Layer
  - A consequence of viscous effects associated with relative motion between a fluid and a surface.
  - A region of the flow characterized by shear stresses and velocity gradients.
  - A region between the surface and the free stream whose thickness δ increases in the flow direction.
  - Why does δ increase in the flow direction?
  - Manifested by a surface shear stress τ_s that provides a drag force, F_D .
  - How does τ_s vary in the flow direction? Why?

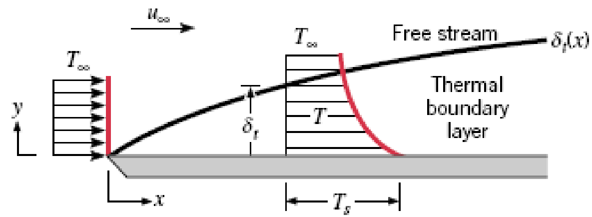


$$\delta \rightarrow \frac{u(y)}{u_\infty} = 0.99$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} \qquad [\text{N/m}^2]$$

$$F_D = \int_{A_s} \tau_s dA_s \qquad [\text{N}]$$

- Thermal Boundary Layer
  - A consequence of heat transfer between the surface and fluid.
  - A region of the flow characterized by temperature gradients and heat fluxes.
  - A region between the surface and the free stream whose thickness δ_t increases in the flow direction.
  - Why does δ_t increase in the flow direction?
  - Manifested by a surface heat flux q_s'' and a convection heat transfer coefficient h.
  - If $\left(T_s - T_\infty\right)$ is constant, how do q_s'' and h vary in the flow direction?



$$\delta_t \rightarrow \frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

$$q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad [\mathrm{W/m^2}]$$

$$h \equiv \frac{-k_f \partial T / \partial y \big|_{y=0}}{T_s - T_\infty} \qquad [\mathrm{W/m^2 \cdot K}]$$

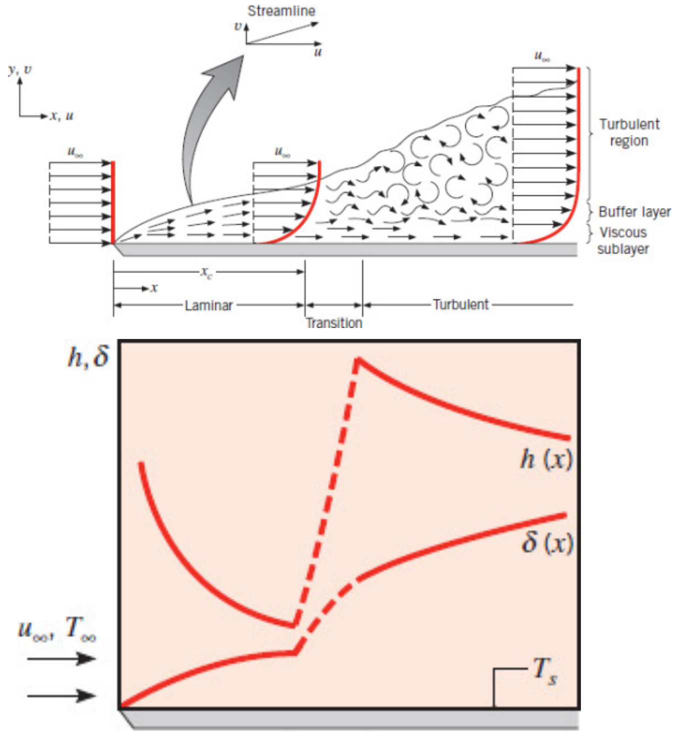
Convection Heat Transfer Coefficient

$$h = \frac{q''}{(T_s - T_\infty)} \quad [\mathrm{W/m^2K}] \qquad h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

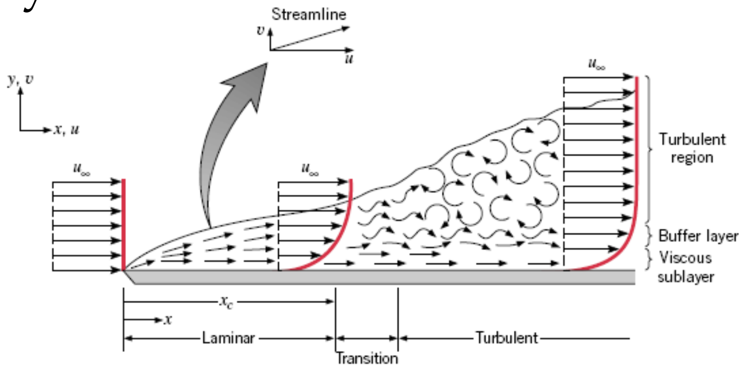
$Nu = \frac{hL}{k_f}$, where L is characteristic length

$Re = \frac{\rho V L}{\mu}$, where L is characteristic length

Laminar and Turbulent Boundary Layers and Heat Transfer



Boundary Layer Transition



- How would you characterize conditions in the laminar region of boundary layer development? In the turbulent region?
- What conditions are associated with transition from laminar to turbulent flow?
- Why is the Reynolds number an appropriate parameter for quantifying transition from laminar to turbulent flow?
- Transition criterion for a flat plate in parallel flow:

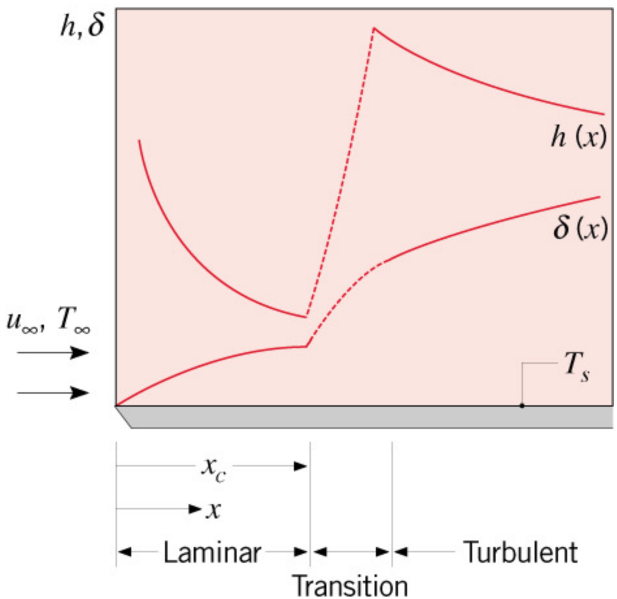
$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} \rightarrow \text{ critical Reynolds number}$$

$x_c \rightarrow$ location at which transition to turbulence begins

$$10^5 \lesssim Re_{x,c} \lesssim 3 \times 10^6$$

$Re_{x,c} = 5 \times 10^5$ is typically used.

- Effect of transition on boundary layer thickness and local convection coefficient:

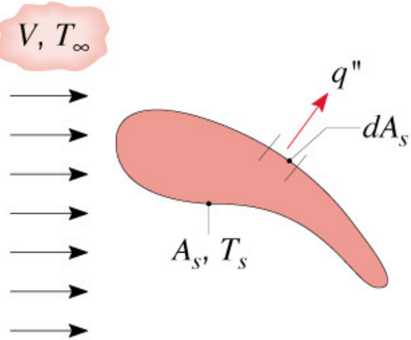


- What may be said about transition if $Re_L < Re_{x,c}$ and $Re_L > Re_{x,c}$?
- Why does the convection coefficient decay in the laminar region?
- Why does it increase significantly with transition to turbulence, despite the increase in the boundary layer thickness?
- Why does the convection coefficient decay in the turbulent region?

Distinction between Local and Average Heat Transfer Coefficients

- Local Heat Flux and Coefficient:

$$q_s'' = h(T_s - T_\infty)$$



- Average Heat Flux and Coefficient for a Uniform Surface Temperature:

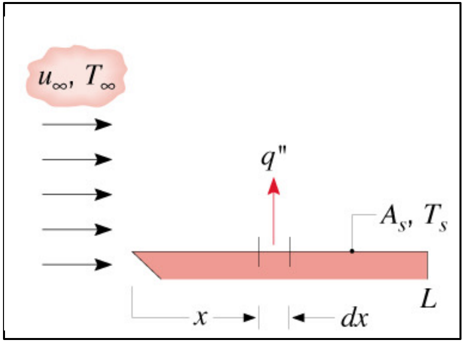
$$q = \bar{h} A_s (T_s - T_\infty)$$

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty) \int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

- For a flat plate in parallel flow:

$$\bar{h} = \frac{1}{L} \int_o^L h dx$$



The Reynolds Analogy

$$\frac{C_f}{2} = Stanton\ Number.\left(= \frac{h}{\rho V c_p} = \frac{Nu}{Re \cdot Pr}\right)$$

valid for $Pr{\sim}1\ and\ Sc{\sim}1$

where $Sc = \frac{\upsilon}{D_{AB}} \sim \frac{\text{momentum}}{\text{mass}}$ diffusivities

$Pr = \frac{\upsilon}{\alpha}$, where υ is kinematic viscosity $(= \mu/\rho)$

The Reynolds Analogy

- Equivalence of dimensionless momentum and energy equations for negligible pressure gradient ($dp^*/dx^* \sim 0$) and $Pr \sim 1$:

$$\underbrace{u^*\frac{\partial u^*}{\partial x^*} + \upsilon^*\frac{\partial u^*}{\partial y^*}}_{\text{Advection terms}} = \underbrace{\frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}}}_{\text{Diffusion}}$$

$$\overbrace{u^*\frac{\partial T^*}{\partial x^*} + \upsilon^*\frac{\partial T^*}{\partial y^*}} = \overbrace{\frac{1}{Re}\frac{\partial^2 T^*}{\partial y^{*2}}}$$

- Hence, for equivalent boundary conditions, the solutions are of the same form:

$$u^* = T^*$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$C_f \frac{Re}{2} = Nu \qquad (6.66)$$

or, with the Stanton number defined as,

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{Re\,Pr}$$

With $Pr = 1$, the Reynolds analogy, which relates important parameters of the velocity and thermal boundary layers, is

$$\frac{C_f}{2} = St \qquad (6.69)$$

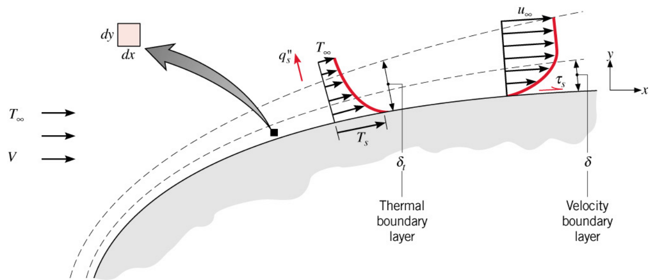
- Modified Reynolds (Chilton-Colburn) Analogy:
  - An empirical result that extends applicability of the Reynolds analogy:

$$\frac{C_f}{2} = St\ Pr^{2/3} \equiv j_H \qquad 0.6 < Pr < 60 \qquad (6.70)$$

Colburn j factor for heat transfer

  - Applicable to laminar flow if $dp^*/dx^* \sim 0$.
  - Generally applicable to turbulent flow without restriction on dp^*/dx^*.

The Boundary Layer Equations



- Consider concurrent velocity and thermal boundary layer development for steady, two-dimensional, incompressible flow with constant fluid properties $\left(\mu, c_p, k\right)$ and negligible body forces.

- Apply conservation of mass, Newton's 2nd Law of Motion and conservation of energy to a differential control volume and invoke the boundary layer approximations.

Velocity Boundary Layer:

$$\frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}, \frac{\partial p}{\partial x} \approx \frac{dp_\infty}{dx}$$

Thermal Boundary Layer:

$$\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$$

- Conservation of Mass:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \qquad (6.27)$$

In the context of flow through a differential control volume, what is the physical significance of the foregoing terms, if each is multiplied by the mass density of the fluid?

- Newton's Second Law of Motion:

x-direction :

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp_\infty}{dx}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (6.28)$$

What is the physical significance of each term in the foregoing equation?

Why can we express the pressure gradient as dp_∞/dx instead of $\partial p/\partial x$?

- Conservation of Energy:

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (6.29)$$

What is the physical significance of each term in the foregoing equation?

What is the second term on the right-hand side called and under what conditions may it be neglected?

Boundary Layer Similarity

- As applied to the boundary layers, the principle of similarity is based on determining similarity parameters that facilitate application of results obtained for a surface experiencing one set of conditions to geometrically similar surfaces experiencing different conditions. (Recall how introduction of the similarity parameters *Bi* and *Fo* permitted generalization of results for transient, one-dimensional conduction).
- Dependent boundary layer variables of interest are:

$$\tau_s \text{ and } q'' \text{ (or } h\text{)}$$

- For a prescribed geometry, the corresponding independent variables are:

Geometrical: Size (L), Location (x,y)
Hydrodynamic: Velocity (V)
Fluid Properties: Hydrodynamic: ρ, μ
Thermal: c_p, k

Hence,

$$u = f\left(x, y, L, V, \rho, \mu\right)$$

$$\tau_s = f\left(x, L, V, \rho, \mu\right)$$

and

$$T = f\left(x, y, L, V, \rho, \mu, c_p, k, T_s, T_\infty\right)$$

$$h = f\left(x, L, V, \rho, \mu, c_p, k, T_s, T_\infty\right)$$

- Key similarity parameters may be inferred by non-dimensionalizing the momentum and energy equations.
- Recast the boundary layer equations by introducing dimensionless forms of the independent and dependent variables.

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L}$$

$$u^* \equiv \frac{u}{V} \qquad \upsilon^* \equiv \frac{\upsilon}{V}$$

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

- Neglecting viscous dissipation, the following normalized forms of the x-momentum and energy equations are obtained:

$$u^* \frac{\partial u^*}{\partial x^*} + \upsilon^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} \qquad (6.35)$$

$$u^* \frac{\partial T^*}{\partial x^*} + \upsilon^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}} \qquad (6.36)$$

$$Re_L \equiv \frac{\rho V L}{\mu} = \frac{VL}{\nu} \rightarrow \text{ the Reynolds Number} \qquad (6.41)$$

$$Pr \equiv \frac{c_p \mu}{k} = \frac{\nu}{\alpha} \rightarrow \text{ the Prandtl Number} \qquad (6.42)$$

How may the Reynolds and Prandtl numbers be interpreted physically?

- For a prescribed geometry,

$$u^* = f\left(x^*, y^*, Re_L\right)$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \left(\frac{\mu V}{L}\right) \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless shear stress, or local friction coefficient, is then

$$C_f \equiv \frac{\tau_s}{\rho V^2 / 2} = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} \qquad (6.45)$$

$$\frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, Re_L\right)$$

$$C_f = \frac{2}{Re_L} f\left(x^*, Re_L\right) \qquad (6.46)$$

What is the functional dependence of the average friction coefficient?

- For a prescribed geometry,

$$T^* = f\left(x^*, y^*, Re_L, Pr\right)$$

$$h = \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty} = -\frac{k_f}{L}\frac{\left(T_\infty - T_s\right)}{\left(T_s - T_\infty\right)}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = +\frac{k_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless local convection coefficient is then

$$Nu \equiv \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, Re_L, Pr\right) \qquad (6.48; 6.49)$$

$Nu \rightarrow$ local Nusselt number

What is the functional dependence of the average Nusselt number?

How does the Nusselt number differ from the Biot number?

The Flat Plate in Parallel Flow

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7.4}$$

Momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \tag{7.5}$$

Energy:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{7.6}$$

The Flat Plate in Parallel Flow

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

- Based on premise that the dimensionless x-velocity component, u / u_∞, and temperature, $T^* \equiv \left[\left(T - T_s\right) / \left(T_\infty - T_s\right)\right]$ can be represented exclusively in terms of a dimensionless similarity parameter

$$\eta \equiv y\left(u_\infty / \nu x\right)^{1/2}$$

- Similarity permits transformation of the partial differential equations associated with the transfer of x-momentum and thermal energy to ordinary differential equations of the form

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \qquad \text{where } \left(u / u_\infty\right) \equiv df / d\eta,$$

and $$\frac{d^2 T^*}{d\eta^2} + \frac{Pr}{2} f \frac{dT^*}{d\eta} = 0$$

The Flat Plate in Parallel Flow

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \qquad \text{where } (u/u_\infty) \equiv df/d\eta,$$

$$\eta \equiv y(u_\infty/\nu x)^{1/2}$$

TABLE 7.1 Flat plate laminar boundary layer functions [3]

$\eta = y\sqrt{\frac{u_\infty}{\nu x}}$	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.4	0.027	0.133	0.331
0.8	0.106	0.265	0.327
1.2	0.238	0.394	0.317
1.6	0.420	0.517	0.297
2.0	0.650	0.630	0.267
2.4	0.922	0.729	0.228
2.8	1.231	0.812	0.184
3.2	1.569	0.876	0.139
3.6	1.930	0.923	0.098
4.0	2.306	0.956	0.064
4.4	2.692	0.976	0.039
4.8	3.085	0.988	0.022
5.2	3.482	0.994	0.011
5.6	3.880	0.997	0.005
6.0	4.280	0.999	0.002
6.4	4.679	1.000	0.001
6.8	5.079	1.000	0.000

The Flat Plate in Parallel Flow

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

Subject to prescribed boundary conditions, numerical solutions to the momentum and energy equations yield the following results for the *x*-component velocity distribution and the temperature distribution in the boundary layer:

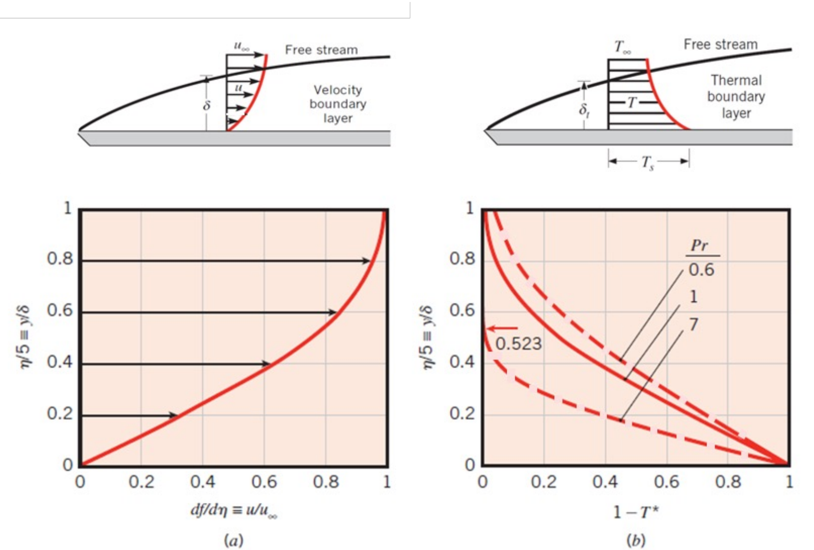


FIGURE 7.4 Similarity solution for laminar flow over an isothermal plate. (*a*) The *x*-component of the velocity. (*b*) Temperature distributions for Pr = 0.6, 1, and 7.

The Flat Plate in Parallel Flow

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

With $u/u_\infty = 0.99$ at $\eta = 5.0$ at the edge of the velocity boundary layer, $\eta \equiv y\left(u_\infty / \nu x\right)^{1/2}$

$$\delta = \frac{5.0}{\left(u_\infty / \nu x\right)^{1/2}} = \frac{5x}{\left(Re_x\right)^{1/2}} \qquad (7.19)$$

With $\tau_s = \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{u_\infty / \nu x} \left.\dfrac{d^2 f}{d\eta^2}\right|_{\eta=0}$

and $\left. d^2 f / d\eta^2 \right|_{\eta=0} = 0.332$,

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\rho u_\infty^2 / 2} = 0.664 Re_x^{-1/2} \qquad (7.20)$$

The Flat Plate in Parallel Flow

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

$$\frac{d^2T^*}{d\eta^2} + \frac{Pr}{2} f \frac{dT^*}{d\eta} = 0$$

With $h_x = q_s''/(T_s - T_\infty) = k\partial T^*/\partial y\big|_{y=0} = k(u_\infty/\nu x)^{1/2} dT^*/d\eta\big|_{\eta=0}$

and $dT^*/d\eta\big|_{\eta=0} = 0.332 \ Pr^{1/3}$ for $Pr > 0.6$,

$$Nu_x = \frac{h_x x}{k} = 0.332 \ Re_x^{1/2} Pr^{1/3} \qquad (7.23)$$

where, $\dfrac{\delta}{\delta_t} = Pr^{1/3}$

The Flat Plate in Parallel Flow

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

- Average Boundary Layer Parameters:

$$\bar{\tau}_{s,x} \equiv \frac{1}{x}\int_0^x \tau_s dx$$

$$\overline{C_{f,x}} = 1.328\ Re_x^{-1/2} \quad (7.29)$$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx$$

$$\overline{Nu}_x = 0.664\ Re_x^{1/2} Pr^{1/3} \quad (7.31)$$

- The effect of variable properties may be considered by evaluating all properties at the film temperature.

$$T_f = \frac{T_s + T_\infty}{2}$$

The Flat Plate in Parallel Flow – Turbulent Flow

- Local Parameters:

Empirical Correlations

$$C_{f,x} = 0.0592\ Re_x^{-1/5} \qquad (7.34)$$

$$Nu_x = 0.0296\ Re_x^{4/5} Pr^{1/3} \qquad (7.36)$$

- Average Parameters:

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{\text{lam}} dx + \int_{x_c}^{L} h_{\text{turb}} dx\right)$$

Substituting expressions for the local coefficients and assuming $Re_{x,c} = 5 \times 10^5$,

$$\bar{C}_{f,L} = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad (7.40)$$

$$\overline{Nu}_L = \left(0.037\ Re_L^{4/5} - 871\right) Pr^{1/3} \qquad (7.38)$$

For $Re_{x,c} = 0$ or $L >> x_c \left(Re_L >> Re_{x,c}\right)$,

$$\bar{C}_{f,L} = 0.074\ Re_L^{-1/5}$$

$$\overline{Nu}_L = 0.037\ Re_L^{4/5} Pr^{1/3}$$

The Empirical Method - Correlations

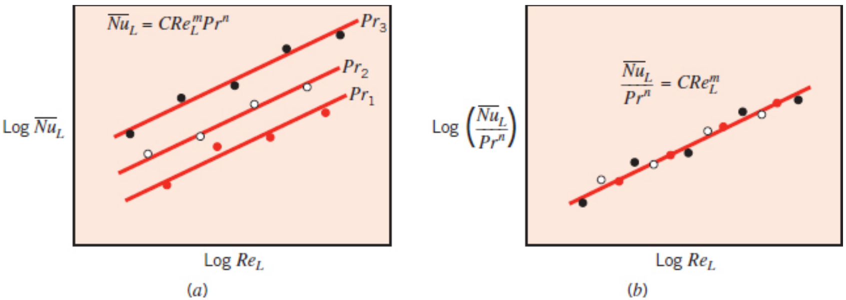
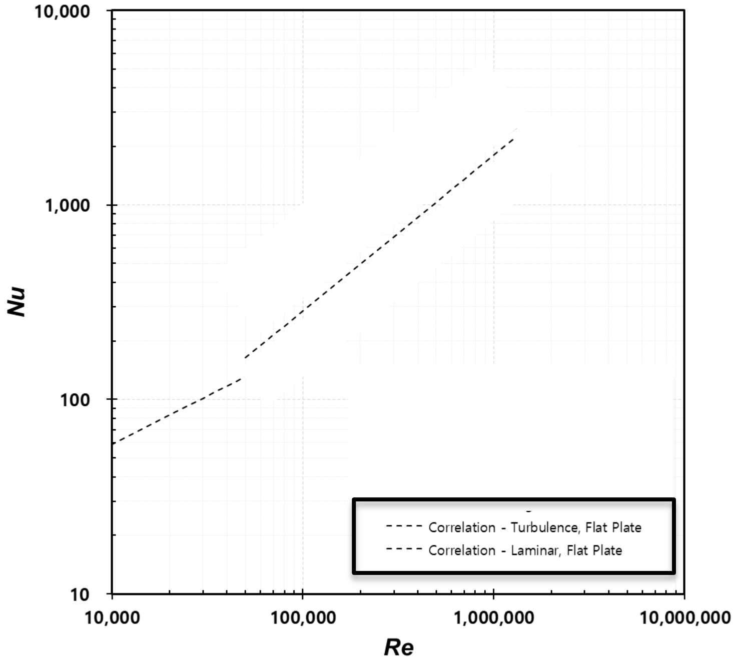


FIGURE 7.2 Dimensionless representation of convection heat transfer measurements.

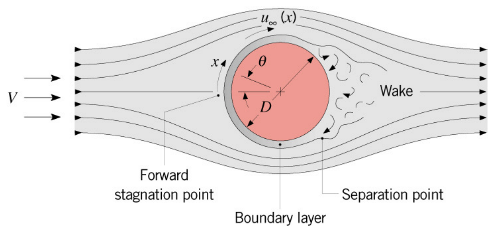
$$\overline{Nu}_L = C \; Re_L^m \; Pr^n$$

The Flat Plate – Nu Correlations



Flow over Cylinders

- Conditions depend on special features of boundary layer development, including onset at a stagnation point and separation, as well as transition to turbulence.



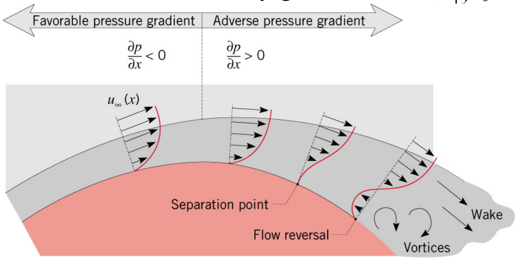
  - Stagnation point: Location of zero velocity $(u_\infty = 0)$ and maximum pressure.

  - Followed by boundary layer development under a favorable pressure gradient $(dp/dx < 0)$ and hence acceleration of the free stream flow $(du_\infty/dx > 0)$.

  - As the rear of the cylinder is approached, the pressure must begin to increase. Hence, there is a minimum in the pressure distribution, *p(x),* after which boundary layer development occurs under the influence of an adverse pressure gradient $(dp/dx > 0,\ du_\infty/dx < 0)$.

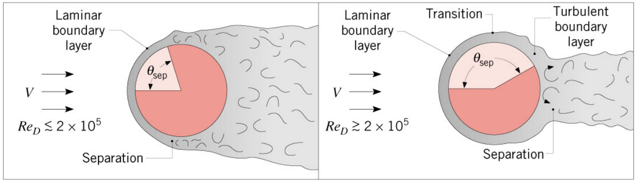
Flow over Cylinders

– Separation occurs when the velocity gradient $du/dy|_{y=0}$ reduces to zero



and is accompanied by flow reversal and a downstream wake.

– Location of separation depends on boundary layer transition.



$$Re_D \equiv \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

Flow over Cylinders

- Force imposed by the flow is due to the combination of *friction* and *form drag.*

The dimensionless form of the drag force is $C_D = \dfrac{F_D}{A_f\left(\rho V^2/2\right)} \rightarrow$ Figure 7.9

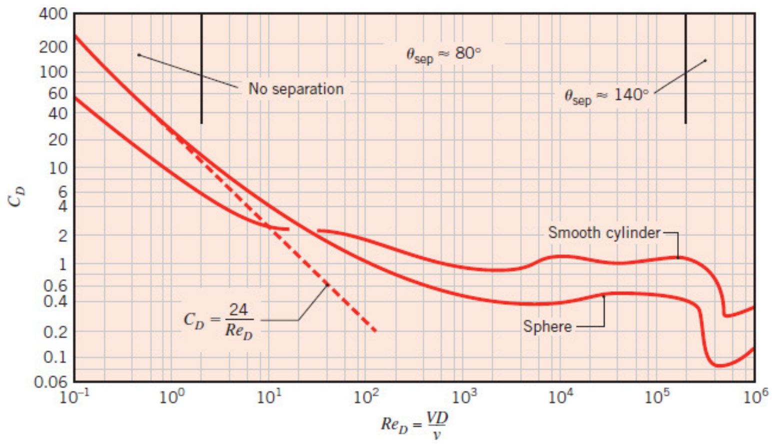


FIGURE 7.9 Drag coefficients for a smooth circular cylinder in cross flow and for a sphere. Boundary layer separation angles are for a cylinder. Based on Schlichting, H., and K. Gersten, *Boundary Layer Theory*, Springer, New York, 2000

Flow over Cylinders

The Average Nusselt Number $\left(\overline{Nu}_D \equiv \bar{h}D / k\right)$:

- Churchill and Bernstein Correlation:

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

- Cylinders of Noncircular Cross Section:

$$\overline{Nu}_D = C\ Re_D^m Pr^{1/3}$$

$$C, m \rightarrow \text{Table 7.3}$$

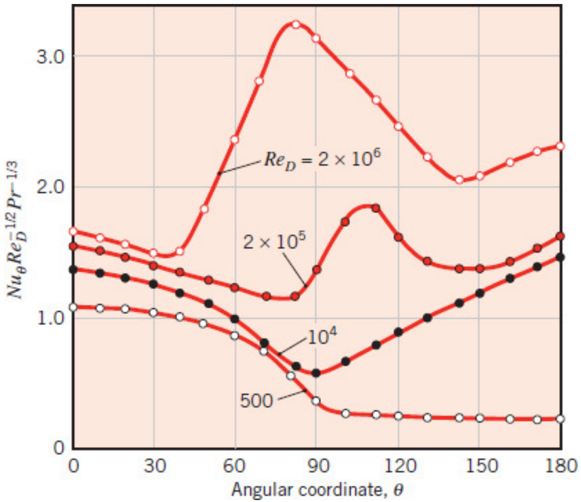
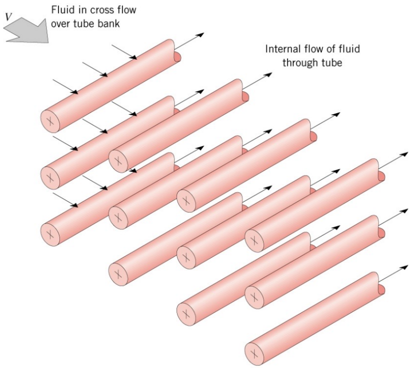


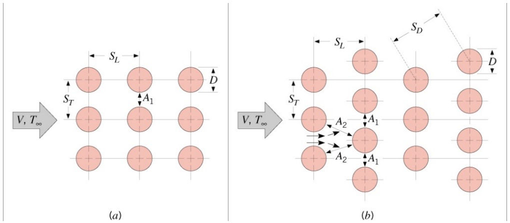
FIGURE 7.10 Local Nusselt number for airflow normal to a circular cylinder. (Adapted with permission from Zukauskas, A., "Convective Heat Transfer in Cross Flow," in S. Kakac, R. K. Shah, and W. Aung, Eds., *Handbook of Single-Phase Convective Heat Transfer*, Wiley, New York, 1987.)

Flow across Tube Banks

- A common geometry for two-fluid heat exchangers.



- Aligned and Staggered Arrays:

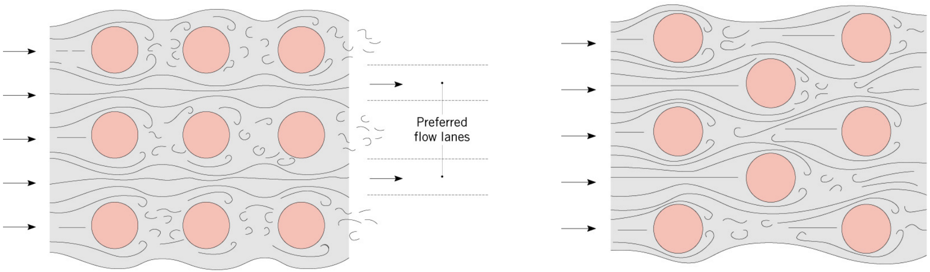


Aligned: $$V_{max} = \frac{S_T}{S_T - D} V$$

Staggered: $$V_{max} = \frac{S_T}{S_T - D} V \quad \text{if} \quad 2\left(S_D - D\right) \geq \left(S_T - D\right)$$

$$\text{or, } V_{max} = \frac{S_T}{2\left(S_D - D\right)} V \quad \text{if} \quad 2\left(S_D - D\right) \leq \left(S_T - D\right)$$

Flow across Tube Banks



- Average Nusselt Number for an Isothermal Array:

$$\overline{Nu}_D = C_2 \left[C_1 Re_{D,\max}^m Pr^{0.36} \left(Pr / Pr_s \right)^{1/4} \right] \qquad (7.58, 7.59)$$

$$C_1, m \rightarrow \text{Table 7.5}$$

$$C_2 \rightarrow \text{Table 7.6}$$

All properties are evaluated at $\left(T_i + T_o\right)/2$ except for Pr_s.

Flow across Tube Banks

- Total Heat Rate (log-mean temp difference):

$$q = \bar{h} A_s \Delta T_{\ell m}$$

$$A_s = N(\pi D L)$$

$$\Delta T_{\ell m} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\dfrac{T_s - T_i}{T_s - T_o}\right)} \qquad (7.62)$$

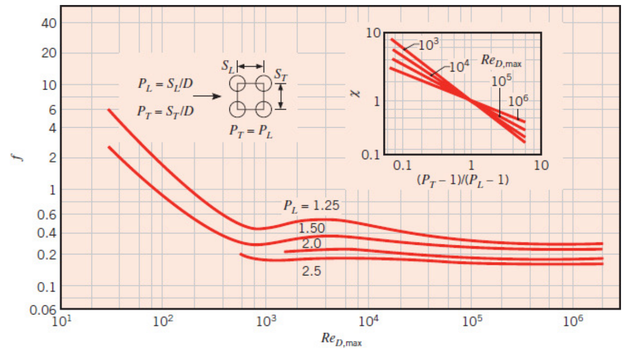
- Fluid Outlet Temperature (T_o) :

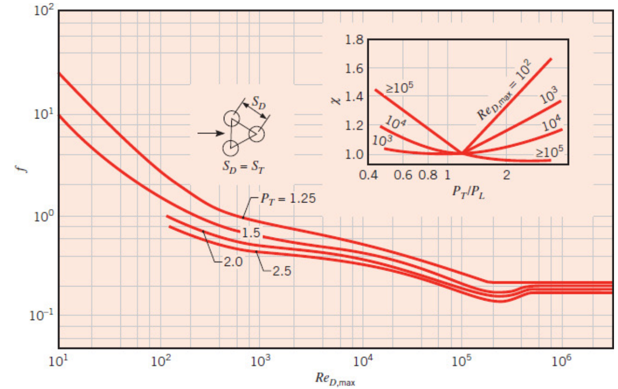
$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right) \qquad (7.63)$$

$$N = N_T \times N_L$$

- Pressure Drop:

$$\Delta p = N_L \chi \left(\frac{\rho V_{max}^2}{2}\right) f \qquad (7.65)$$

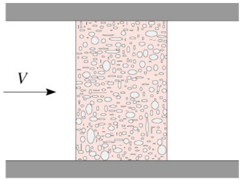




Flow over Spheres, and Packed Beds

- Flow over a rigid sphere
  - Boundary layer development is similar to that for flow over a cylinder, involving transition and separation.
  - $$\overline{Nu}_D = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Pr^{0.4} \left(\mu / \mu_s\right)^{1/4} \quad (7.56)$$
  - All properties are evaluated at T_∞, except μ_s at T_s.

- Gas Flow through a Packed Bed



  - Flow is characterized by tortuous paths through a bed of fixed particles.
  - Large surface area per unit volume renders configuration desirable for the transfer and storage of thermal energy.

Flow over Spheres, and Packed Beds

– For a packed bed of spheres:

$$\varepsilon \bar{j}_H = 2.06 \, Re_D^{-0.575} \tag{7.81}$$

$\varepsilon \rightarrow$ void fraction ($0.3 < \varepsilon < 0.5$)

– $q = \bar{h} A_{p,t} \Delta T_{\ell m}$

$A_{p,t} \rightarrow$ total surface area of particles

– $$\frac{T_s - T_o}{T_s - T_i} = \exp\left(- \frac{\bar{h} A_{p,t}}{\rho V A_{c,b} c_p} \right) \tag{7.83}$$

$A_{c,b} \rightarrow$ cross-sectional area of bed

Modified Reynolds Analogy (Chapter 6)

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{Re\,Pr} \tag{6.67}$$

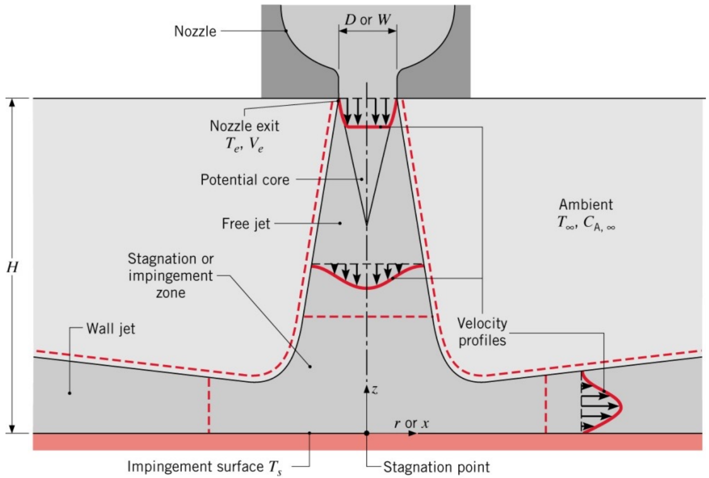
The validity of the boundary layer approximations, the accuracy of Equation 6.69 depends on having Pr and $Sc \approx 1$ and $dp^*/dx^* \approx 0$.

However, it has been shown that the analogy may be applied over a wide range of Pr and Sc, if certain corrections are added. In particular the *modified Reynolds*, or *Chilton–Colburn, analogies* [13, 14], have the form

$$\frac{C_f}{2} = St\,Pr^{2/3} \equiv j_H \quad 0.6 < Pr < 60 \tag{6.70}$$

Jet Impingement

Characterized by large convection coefficients and used for cooling and heating in numerous manufacturing, electronic and aeronautic applications.



- Mixing and velocity profile development in the free jet.
- Stagnation point and zone.
- Velocity profile development in the wall jet.

Jet Impingement

Average Nusselt number: $\overline{Nu} = \frac{\bar{h}D_h}{k} = f\left(Re, Pr, A_r, H / D_h\right)$

$Re = \frac{V_e D_h}{\nu}$, A_r from Fig. 7.18

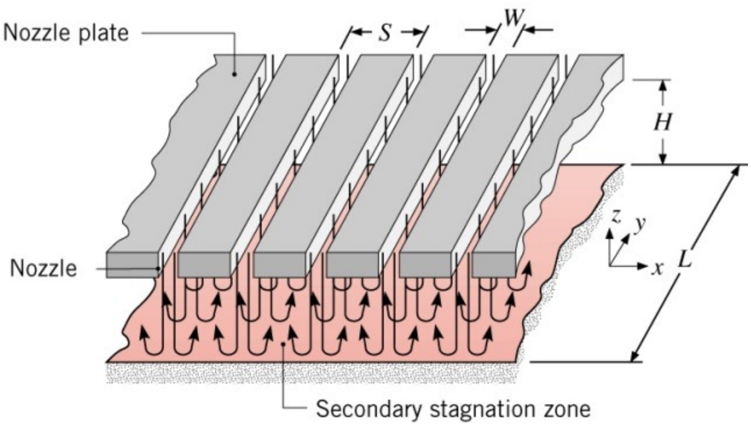
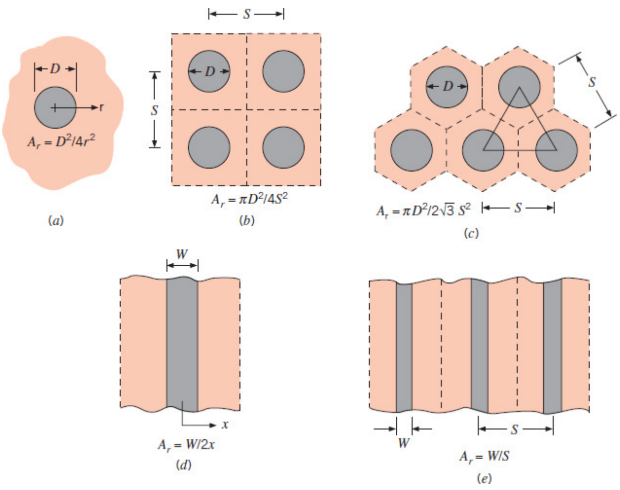
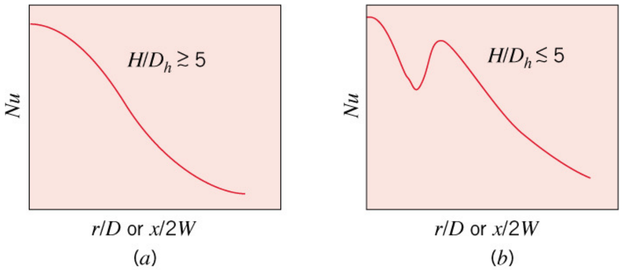


FIGURE 7.18 Plan view of pertinent geometrical features for (*a*) single round jet, (*b*) in-line array of round jets, (*c*) staggered array of round jets, (*d*) single slot jet, and (*e*) array of slot jets.



Jet Impingement



Round Nozzles Having assessed data from several sources, Martin [22] recommends the following correlation for a *single round nozzle* ($A_r = D^2/4r^2$)

$$\frac{\overline{Nu}}{Pr^{0.42}} = G\left(A_r, \frac{H}{D}\right)\left[2\ Re^{1/2}\left(1 + 0.005\ Re^{0.55}\right)^{1/2}\right] \tag{7.71}$$

where

$$G = 2A_r^{1/2}\frac{1 - 2.2A_r^{1/2}}{1 + 0.2(H/D - 6)A_r^{1/2}} \tag{7.72}$$

The ranges of validity are

$$\begin{bmatrix} 2000 \lesssim Re \lesssim 400{,}000 \\ 2 \lesssim H/D \lesssim 12 \\ 0.004 \lesssim A_r \lesssim 0.04 \end{bmatrix}$$

For $A_r \gtrsim 0.04$, results for $\overline{Nu}$ are available in graphical form [22].

Chapter 8 Internal Flow

Convection Heat Transfer Coefficient – Internal Flow

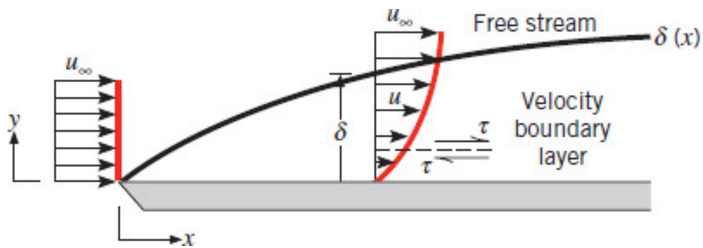
$$h = \frac{q''}{(T_s - T_\infty)} \quad [\mathrm{W/m^2K}] \qquad h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

$$Nu = \frac{hD_h}{k_f}$$, where D_h is Hydraulic Diameter

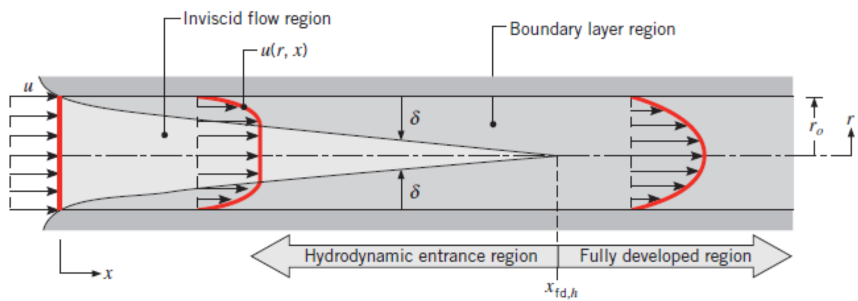
$$Re = \frac{\rho V_m D_h}{\mu}$$, where D_h is Hydraulic Diameter

Velocity boundary layer development

Flat plate
(External flow)



Circular tube
(Internal flow)



The Mean Velocity and Velocity Profile

- Absence of well-defined free stream conditions, as in external flow, and hence a reference velocity (u_∞) or temperature (T_∞), dictates the use of a cross-sectional mean velocity (u_m) and temperature (T_m) for internal flow.

- Linkage of mean velocity to mass flow rate:

$$\dot{m} = \rho u_m A_c$$

or,

$$\dot{m} = \int_{A_c} \rho u(r,x)\, dA_c$$

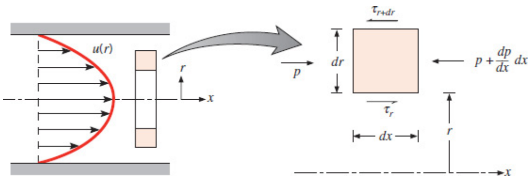
Hence,

$$u_m = \frac{\int_{A_c} \rho u(r,x)\, dA_c}{\rho A_c}$$

For *incompressible flow* in a *circular tube* of radius r_o,

$$u_m = \frac{2\pi\rho}{\rho\pi r_0^2}\int_0^{r_0} u(r,x) r dr \Rightarrow u_m = \frac{2}{r_o^2}\int_0^{r_o} u(r,x) r\, dr$$

- Laminar velocity profile in fully developed region



$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right) r_0^2 \left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_0}\right)^2\right] \qquad (8.15)$$

Flow Characteristics – Internal Flow

- Entry lengths depend on whether the flow is laminar or turbulent, which, in turn, depends on Reynolds number.

$$Re_D \equiv \frac{\rho u_m D_h}{\mu}$$

The hydraulic diameter is defined as

$$D_h \equiv \frac{4A_c}{P}$$

in which case,

$$Re_D \equiv \frac{\rho u_m D_h}{\mu} = \frac{4\dot{m}}{P\mu}$$

For a circular tube,

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{4\dot{m}}{\pi D\mu}$$

- Onset of turbulence occurs at a critical Reynolds number of

$$Re_{D,c} \approx 2300$$

- Fully turbulent conditions exist for

$$Re_D \approx 10{,}000$$

- Hydrodynamic Entry Length

Laminar Flow: $\left(x_{fd,h} / D\right) \approx 0.05\, Re_D$

Turbulent Flow: $10 < \left(x_{fd,h} / D\right) < 60$

- Thermal Entry Length

Laminar Flow: $\left(x_{fd,t} / D\right) \approx 0.05\, Re_D Pr$

Turbulent Flow: $10 < \left(x_{fd,t} / D\right) < 60$

Flow Characteristics – Internal Flow

- Assuming steady flow and constant properties, hydrodynamic conditions, including the velocity profile, are invariant in the fully developed region.

- The pressure drop may be determined from knowledge of the friction factor *f*, where,

$$f \equiv -\frac{(dp/dx)D}{\rho u_m^2/2}$$

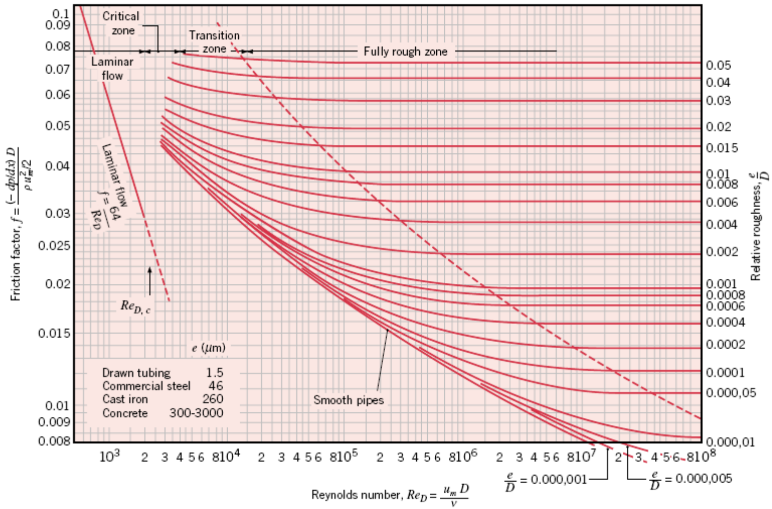
Laminar flow in a circular tube:

$$f = \frac{64}{Re_D} \qquad (8.19)$$

Turbulent flow in a smooth circular tube:

$$f = (0.790 \ln Re_D - 1.64)^{-2} \qquad (8.21)$$

Flow Characteristics – Internal Flow



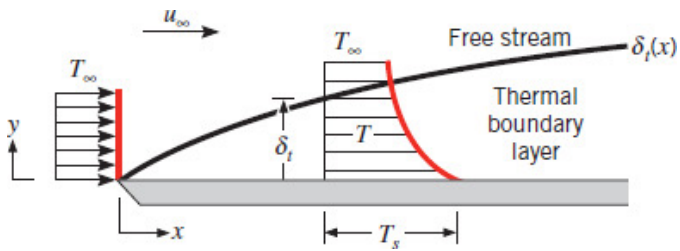
$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{e/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right] \qquad (8.20)$$

Pressure drop for fully developed flow from x_1 to x_2: and power requirement

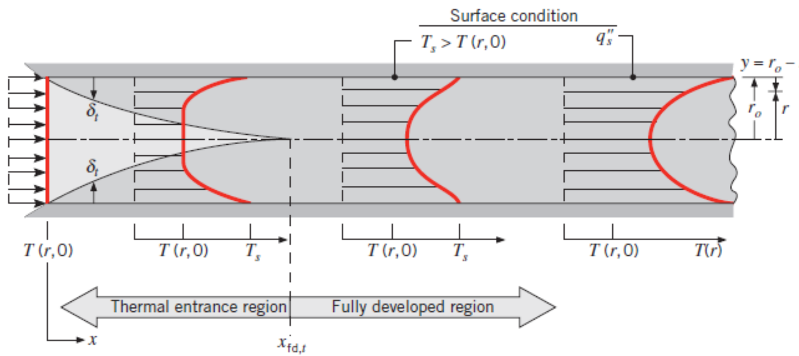
$$\Delta p = p_1 - p_2 = f\,\frac{\rho u_m^2}{2D}\left(x_2 - x_1\right) \qquad P = \Delta p\,\dot{\forall} = \frac{\Delta p\,\dot{m}}{\rho}$$

Thermal boundary layer development

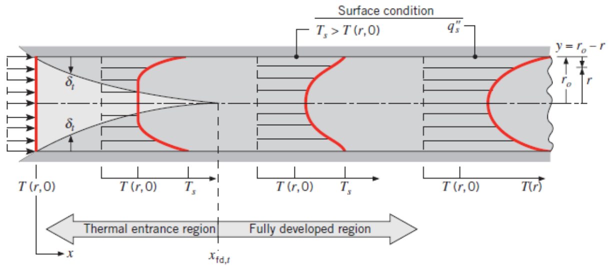
Flat plate
(External flow)



Circular tube
(Internal flow)



Thermal boundary layer development



– Thermal boundary layer develops on surface of tube and thickens with increasing x.

– Isothermal core shrinks as boundary layer grows.

– Subsequent to boundary layer merger, dimensionless forms of the temperature profile (for T_s and q_s'') become independent of x.
→ Conditions are then said to be thermally fully developed.

Determination of the Mean Temperature

- Linkage of mean temperature to thermal energy transport associated with flow through a cross section:

$$q = \dot{m} c_p (T_{out} - T_{in})$$

$$q = \int_{A_c} \rho u c_p T \, dA_c = \dot{m} c_p T_m$$

Hence,

$$T_m = \frac{\int_{A_c} \rho u c_p T \, dA_c}{\dot{m} c_p}$$

- For incompressible, constant-property flow in a circular tube,

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} u(x,r) T(x,r) r \, dr$$

- Newton's law of cooling for the Local Heat Flux:

$$q_s'' = h(T_s - T_m) \qquad (8.27)$$

Determination of the Mean Temperature

- Requirement for fully developed thermal conditions:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]_{fd,t}=0$$

- Effect on the local convection coefficient:

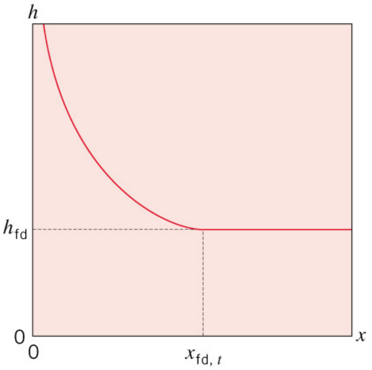
$$\frac{\partial}{\partial r}\left(\frac{T_s-T}{T_s-T_m}\right)\bigg|_{r=r_o}=\frac{-\partial T/\partial r|_{r=r_o}}{T_s-T_m}\neq f(x)$$

Hence, assuming constant properties,

$$\frac{q_s''/k}{T_s-T_m}=\frac{h}{k}\neq f(x)$$

$$h\neq f(x)$$

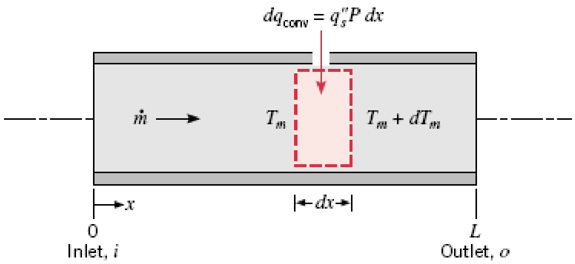
Variation of h in entrance and fully developed regions:



Determination of the Mean Temperature

- Determination of $T_m(x)$ is an essential feature of an internal flow analysis.

  Determination begins with an energy balance for a differential control volume.



$$dq_{\text{conv}} = \dot{m}c_p\left[\left(T_m + dT_m\right) - T_m\right] = \dot{m}c_p dT_m$$

Integrating from the tube inlet to outlet,

$$q_{\text{conv}} = \dot{m}\,c_p\left(T_{m,o} - T_{m,i}\right) \qquad (1)$$

Determination of the Mean Temperature

A differential equation from which $T_m(x)$ may be determined is obtained by substituting for $dq_{\text{conv}} = q_s''(P\,dx) = h(T_s - T_m)P\,dx$.

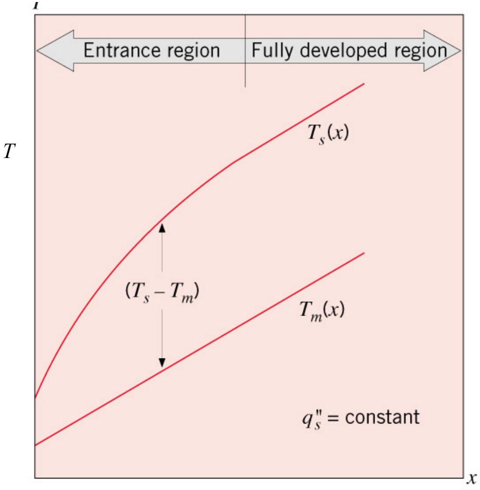
$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}\,c_p} = \frac{P}{\dot{m}\,c_p}h(T_s - T_m) \qquad (2)$$

- Special Case: Uniform Surface Heat Flux

Since total heat rate: $q_{\text{conv}} = q_s''\,PL$

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}\,c_p} \neq f(x)$$

$$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}\,c_p}x$$



Determination of the Mean Temperature

- Special Case: Uniform Surface Temperature

From Eq. (2), with $\Delta T \equiv T_s - T_m$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\,\Delta T$$

Integrating from x=0 to any downstream location,

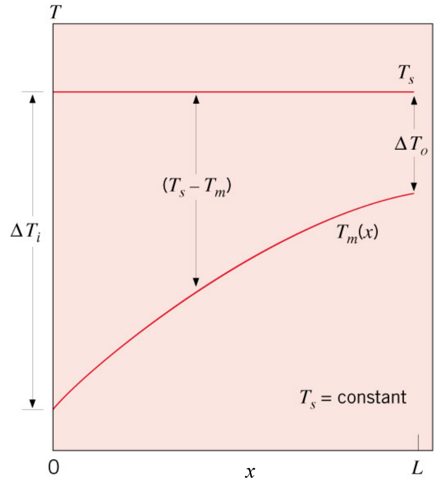
$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}_x\right)$$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x\,dx$$

Overall Conditions:

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right) = \exp\left(-\frac{\bar{h}A_s}{\dot{m}c_p}\right)$$

⇨



$$q_{conv} = \bar{h}A_s\,\Delta T_{\ell m}$$

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}$$

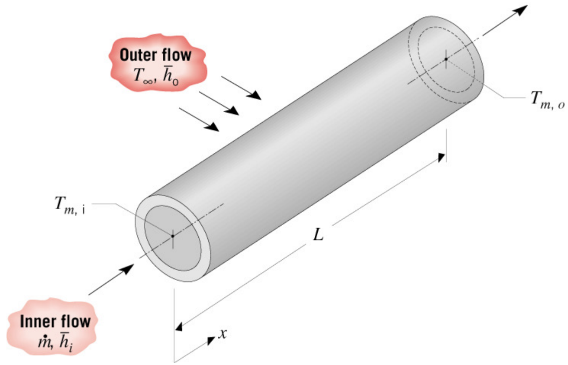
Determination of the Mean Temperature

- Special Case: Uniform *External* Fluid Temperature

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}\,c_p}\right) = \exp\left(-\frac{1}{\dot{m}\,c_p R_{\text{tot}}}\right)$$

$$q = \bar{U}A_s\,\Delta T_{\ell m} = \frac{\Delta T_{\ell m}}{R_{\text{tot}}}$$

$\Delta T_{\ell m} \rightarrow$ Eq. (3) with T_s replaced by T_∞.



Note: Replacement of T_∞ by $T_{s,o}$ if outer surface temperature is uniform.

Heat Transfer Characteristics – Internal Flow

Fully developed region

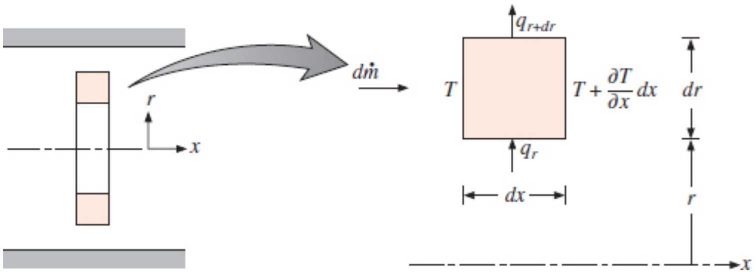


FIGURE 8.9 Thermal energy balance on a differential element for laminar, fully developed flow in a circular tube.

$$q_r - q_{r+dr} = (d\dot{m})c_p\left[\left(T + \frac{\partial T}{\partial x}dx\right) - T\right] \quad (8.47a)$$

or

$$(d\dot{m})c_p\frac{\partial T}{\partial x}dx = q_r - \left(q_r + \frac{\partial q_r}{\partial r}dr\right) = -\frac{\partial q_r}{\partial r}dr \quad (8.47b)$$

Heat Transfer Characteristics – Internal Flow

Fully developed region

$$(d\dot{m})c_p \frac{\partial T}{\partial x} dx = q_r - \left(q_r + \frac{\partial q_r}{\partial r} dr \right) = - \frac{\partial q_r}{\partial r} dr \qquad (8.47b)$$

The differential mass flow rate in the axial direction is $d\dot{m} = \rho u 2\pi r\, dr$, and the radial heat transfer rate is $q_r = -k(\partial T/\partial r)2\pi r dx$. If we assume constant properties, Equation 8.47b becomes

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r} \right) \qquad (8.48)$$

For laminar, fully developed conditions with uniform surface heat flux

Now, Applying (8.15) and (8.32) into (8.48)

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right] \qquad (8.15)$$

$$\left.\frac{\partial T}{\partial x}\right|_{\text{fd},t} = \left.\frac{dT_m}{dx}\right|_{\text{fd},t} \qquad q_s'' = \text{constant} \qquad (8.32)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r} \right) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx} \right)\left[1 - \left(\frac{r}{r_o}\right)^2\right] \qquad q_s'' = \text{constant} \qquad (8.49)$$

where $T_m(x)$ varies linearly with x and $(2u_m/\alpha)(dT_m/dx)$ is a constant.

Heat Transfer Characteristics – Internal Flow

Fully developed region

Separating variables and integrating twice, we obtain an expression for the radial temperature distribution:

$$T(r,x) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{r^2}{4} - \frac{r^4}{16r_0^2}\right] + C_1 \ \ln\ r + C_2$$

where, C_1=0 for T=finite at r=0, and $C_2 = T_s(x) - \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left(\frac{3r_o^2}{16}\right)$

since T=T_s at r=r_0

Accordingly, for the fully developed region with constant surface heat flux, the temperature profile is of the form

$$T(r,x) = T_s(x) - \frac{2u_m r_o^2}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{r_o}\right)^4 - \frac{1}{4}\left(\frac{r}{r_o}\right)^2\right] \quad (8.50)$$

Heat Transfer Characteristics – Internal Flow

Fully developed region

For laminar, fully developed conditions with uniform surface heat flux

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right] \quad (8.15)$$

$$T(r,x) = T_s(x) - \frac{2u_m r_o^2}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{r_o}\right)^4 - \frac{1}{4}\left(\frac{r}{r_o}\right)^2\right] \quad (8.50)$$

To get the mean temperature, apply (8.15) and (8.50) into (8.26)

$$T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} uTr\,dr \quad (8.26)$$

$$T_m(x) = T_s(x) - \frac{11}{48}\left(\frac{u_m r_o^2}{\alpha}\right)\left(\frac{dT_m}{dx}\right) \quad (8.51)$$

Since, $\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}c_p} \neq f(x)$ (8.39)

$$T_m(x) - T_s(x) = -\frac{11}{48}\frac{q_s'' D}{k} \quad (8.52)$$

From, $q_s'' = h(T_s - T_m)$ (8.27) ➡ $h = \frac{48}{11}\left(\frac{k}{D}\right)$

$$Nu_D \equiv \frac{hD}{k} = 4.36 \quad q_s'' = \text{constant} \quad (8.53)$$

Heat Transfer Characteristics – Internal Flow

Fully developed region

For laminar, fully developed conditions with a constant surface temperature

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right] \qquad (8.15)$$

$$\left.\frac{\partial T}{\partial x}\right|_{\mathrm{fd},t} = \frac{(T_s - T)}{(T_s - T_m)} \left.\frac{dT_m}{dx}\right|_{\mathrm{fd},t} \qquad T_s = \text{constant} \qquad (8.33)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[1 - \left(\frac{r}{r_o}\right)^2\right]\frac{T_s - T}{T_s - T_m} \qquad T_s = \text{constant} \qquad (8.54)$$

$$Nu_D = 3.66 \quad T_s = \text{constant} \qquad (8.55)$$

Heat Transfer Characteristics – Internal Flow

Fully developed region

- Laminar Flow in a Circular Tube:

  The local Nusselt number is constant throughout the fully developed region, but its value depends on the surface thermal condition.

  – Uniform Surface Heat Flux (q_s'') :

  $$Nu_D = \frac{hD}{k} = 4.36 \qquad (8.53)$$

  – Uniform Surface Temperature (T_s) :

  $$Nu_D = \frac{hD}{k} = 3.66 \qquad (8.55)$$

- Turbulent Flow in a Circular Tube:

  – For a smooth surface and fully turbulent conditions $(Re_D > 10{,}000)$, the Dittus – Boelter equation may be used as a first approximation:

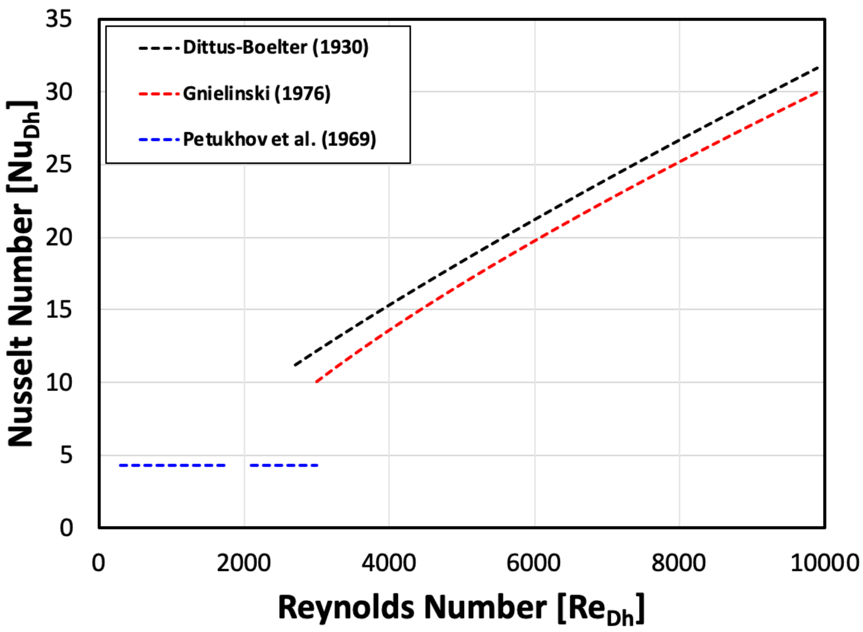
  $$Nu_D = 0.023 Re_D^{4/5} Pr^n \qquad \begin{cases} n = 0.3 & (T_s < T_m) \\ n = 0.4 & (T_s > T_m) \end{cases} \qquad (8.60)$$

  – The effects of wall roughness and transitional flow conditions $(Re_D > 3000)$ may be considered by using the Gnielinski correlation:

  $$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}\left(Pr^{2/3} - 1\right)} \qquad (8.62)$$

Heat Transfer Characteristics – Internal Flow

Fully developed region



Heat Transfer Characteristics – Internal Flow

Fully developed region

- Noncircular Tubes:
  - Use of hydraulic diameter as characteristic length:

$$D_h \equiv \frac{4A_c}{P}$$

  - Since the local convection coefficient varies around the periphery of a tube, approaching zero at its corners, correlations for the fully developed region are associated with convection coefficients averaged over the periphery of the tube.

  - Laminar Flow:

    The local Nusselt number is a constant whose value (Table 8.1) depends on the surface thermal condition $\left(T_s \text{ or } q_s''\right)$ and the duct aspect ratio.

  - Turbulent Flow:

    As a first approximation, the Dittus-Boelter or Gnielinski correlation may be used with the hydraulic diameter, irrespective of the surface thermal condition.

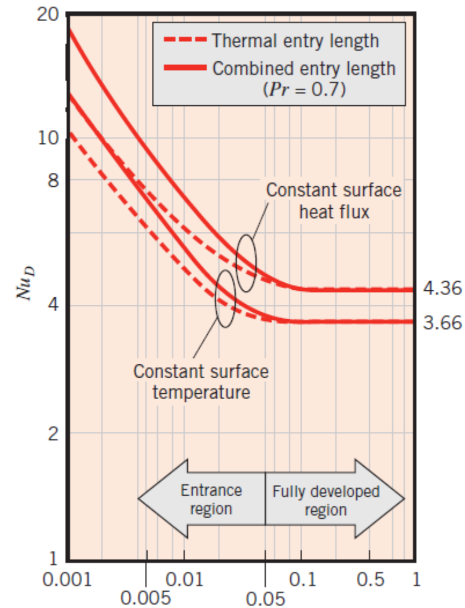
Heat Transfer Characteristics – Internal Flow

Entry region

- The manner in which the Nusselt number decays from inlet to fully developed conditions for laminar flow depends on the nature of thermal and velocity boundary layer development in the entry region, as well as the surface thermal condition.

  – Combined Entry Length:

    Thermal and velocity boundary layers develop concurrently from uniform profiles at the inlet.



$$\frac{x/D}{Re_D Pr} = Gz_D^{-1}$$ (Gz_D: Graetz number)

Heat Transfer Characteristics – Internal Flow

Entry region

– Thermal Entry Length:

Velocity profile is fully developed at the inlet, and boundary layer development in the entry region is restricted to thermal effects. Such a condition may also be assumed to be a good approximation for a uniform inlet velocity profile if $Pr >> 1$.

- Average Nusselt Number for Laminar Flow in a Circular Tube with Uniform Surface Temperature:

  – Combined Entry Length (Baehr and Stephan):

$$\overline{Nu}_D = \frac{\dfrac{3.66}{\tanh\left[2.264\ Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}\right]} + 0.0499\ Gz_D \tanh(Gz_D^{-1})}{\tanh\left(2.432 Pr^{1/6} Gz_D^{-1/6}\right)} \qquad (8.58)$$

  – Thermal Entry Length (Hausen):

$$\overline{Nu}_D = 3.66 + \frac{0.0668 Gz_D}{1 + 0.04 Gz_D^{2/3}} \qquad (8.57)$$

Heat Transfer Characteristics – Internal Flow

Entry region

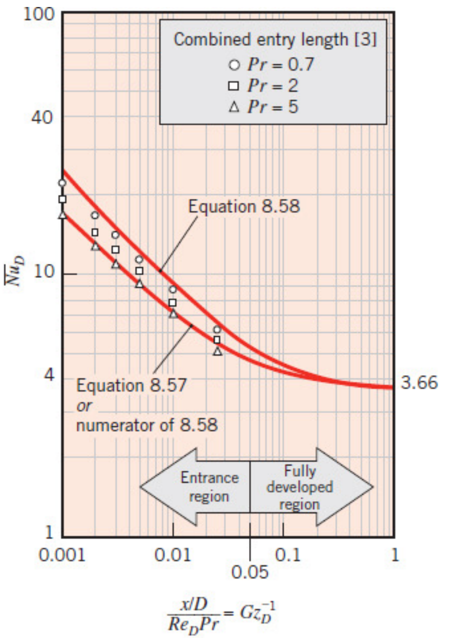
- Average Nusselt Number for Turbulent Flow in a Circular Tube :
  - Effects of entry and surface thermal conditions are less pronounced for turbulent flow and can often be neglected.
  - For long tubes $(L/D > 60)$:

$$\overline{Nu}_D \approx Nu_{D,\text{fd}}$$

  - For short tubes $(L/D < 60)$:

$$\frac{\overline{Nu}_D}{Nu_{D,\text{fd}}} \approx 1 + \frac{C}{(L/D)^m}$$

$$C \approx 1$$
$$m \approx 2/3$$



(b)

Heat Transfer Characteristics – Noncircular tube

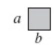
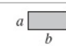
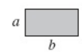
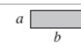
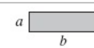
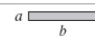
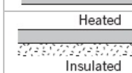
- Noncircular Tubes:
  - Laminar Flow:

    $\overline{Nu}_{D_h}$ depends strongly on aspect ratio, as well as entry region and surface thermal conditions.

  - Turbulent Flow:

    As a first approximation, correlations for a circular tube may be used with D replaced by D_h .

TABLE 8.1 **Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section**

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	(Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
	∞	5.39	4.86	96
	—	3.11	2.49	53

Heat Transfer Characteristics – Noncircular tube

- Temperature-Dependent Properties:

  - When determining $\overline{Nu}_D$ for any tube geometry or flow condition, all properties are to be evaluated at

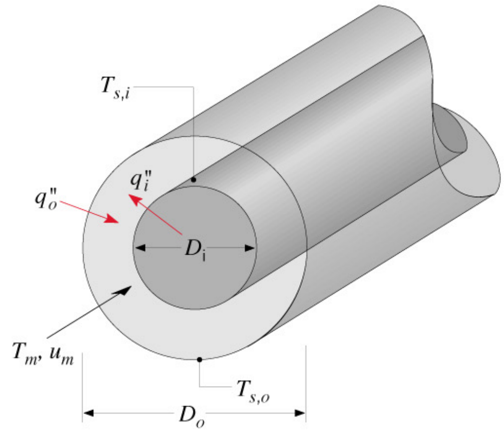
$$\overline{T}_m \equiv \left(T_{m,i} + T_{m,o}\right)/2$$

  - When differences between T_m and T_s correspond to large property variations, the Nusselt numbers for laminar flow of a liquid can be corrected as

$$\frac{Nu_{D,c}}{Nu_D} = \frac{\overline{Nu}_{D,c}}{\overline{Nu}_D} = \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

where the corrected Nusselt numbers are denoted by subscript c.

Heat Transfer Characteristics – Concentric tube



- Fluid flow through region formed by concentric tubes.

- Convection heat transfer may be from or to inner surface of outer tube and outer surface of inner tube.

- Surface thermal conditions may be characterized by uniform temperature $\left(T_{s,i}, T_{s,o}\right)$ or uniform heat flux $\left(q_i'', q_o''\right)$.

- Convection coefficients are associated with each surface, where

$$q_i'' = h_i\left(T_{s,i} - T_m\right)$$

$$q_o'' = h_o\left(T_{s,o} - T_m\right)$$

Heat Transfer Characteristics – Concentric tube

$$Nu_i \equiv \frac{h_i D_h}{k} \qquad Nu_o \equiv \frac{h_o D_h}{k}$$

$$D_h = D_o - D_i$$

- Fully Developed Laminar Flow

  Nusselt numbers depend on D_i / D_o and surface thermal conditions (Tables 8.2, 8.3)

- Fully Developed Turbulent Flow

  Correlations for a circular tube may be used with D replaced by D_h .

TABLE 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	*Comments*
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
≈1.00	4.86	4.86	See Table 8.1, $b/a \to \infty$

TABLE 8.3 Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces

D_i/D_o	Nu_{ii}	Nu_{oo}	θ_i^*	θ_o^*
0	—	4.364[a]	∞	0
0.05	17.81	4.792	2.18	0.0294
0.10	11.91	4.834	1.383	0.0562
0.20	8.499	4.883	0.905	0.1041
0.40	6.583	4.979	0.603	0.1823
0.60	5.912	5.099	0.473	0.2455
0.80	5.58	5.24	0.401	0.299
1.00	5.385	5.385[b]	0.346	0.346

Heat Transfer Enhancement

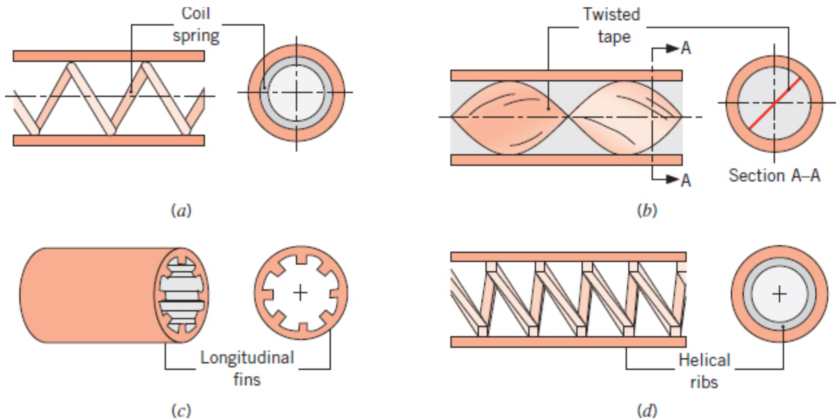


FIGURE 8.12 Internal flow heat transfer enhancement schemes: (*a*) longitudinal section and end view of coil-spring wire insert, (*b*) longitudinal section and cross-sectional view of twisted tape insert, (*c*) cut-away section and end view of longitudinal fins, and (*d*) longitudinal section and end view of helical ribs.

Heat Transfer Enhancement

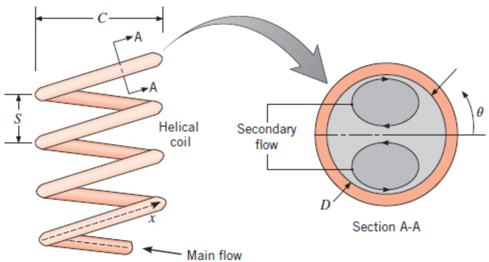


FIGURE 8.13 Schematic of helically coiled tube and secondary flow in enlarged cross-sectional view.

$$Re_{D,c,h} = Re_{D,c}\left[1 + 12(D/C)^{0.5}\right] \tag{8.74}$$

$$f = \frac{27}{Re_D^{0.725}}(D/C)^{0.1375} \quad 30 \lesssim Re_D\,(D/C)^{1/2} \lesssim 300 \tag{8.75a}$$

$$f = \frac{7.2}{Re_D^{0.5}}(D/C)^{0.25} \quad 300 \lesssim Re_D\,(D/C)^{1/2} \tag{8.75b}$$

$$Nu_D = \left[\left(3.66 + \frac{4.343}{a}\right)^3 + 1.158\left(\frac{Re_D\,(D/C)^{1/2}}{b}\right)^{3/2}\right]^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \tag{8.76}$$

where

$$a = \left(1 + \frac{927(C/D)}{Re_D^2\ Pr}\right) \quad \text{and} \quad b = 1 + \frac{0.477}{Pr} \tag{8.77a,b}$$

$$\begin{bmatrix} 0.005 \lesssim Pr \lesssim 1600 \\ 1 \lesssim Re_D\,(D/C)^{1/2} \lesssim 1000 \end{bmatrix}$$