ME 3304 Heat Transfer

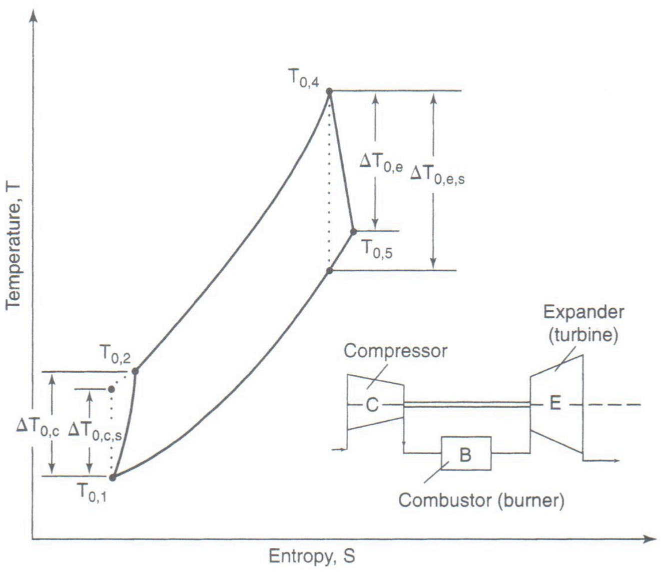
Lecture Note (6) Heat Exchangers (Chapter 11)

Prof. Changmin Son
changminson@vt.edu

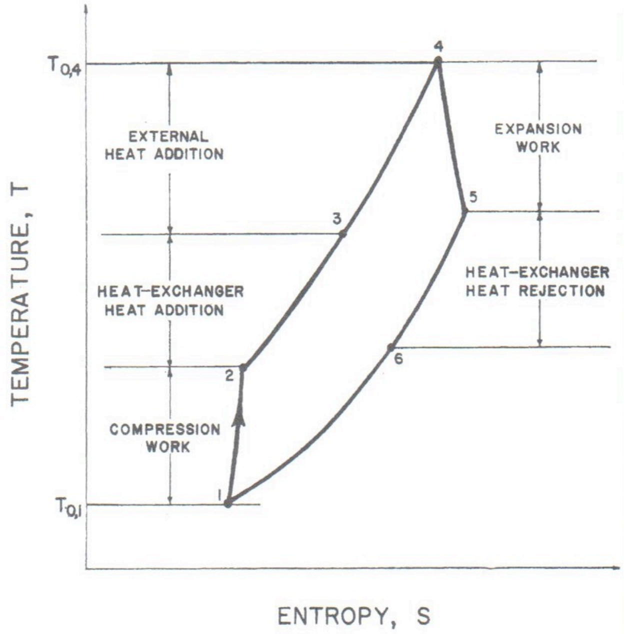
Why Heat Exchanger?

Brayton (Gas Turbine) Cycle

Non-isentropic Process

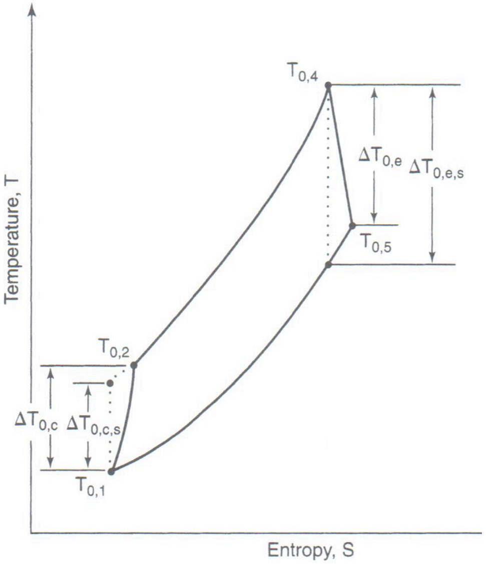


Possible Heat Exchanger Cycles

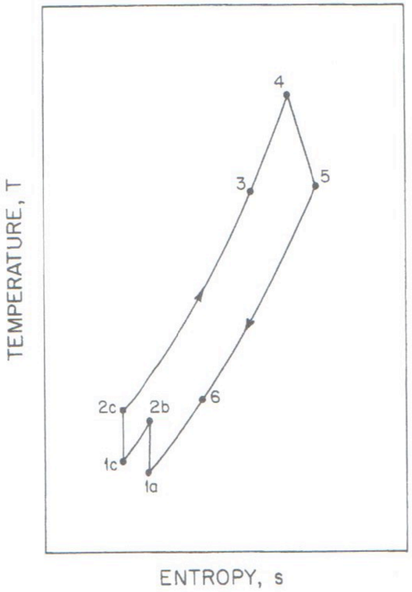


Simple and Heat Exchanger Cycles

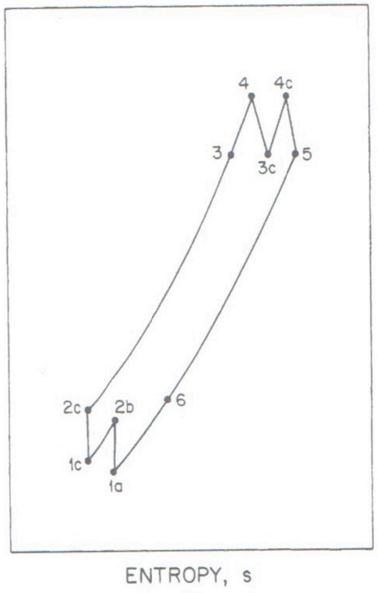
Intercooled and Reheated Cycles



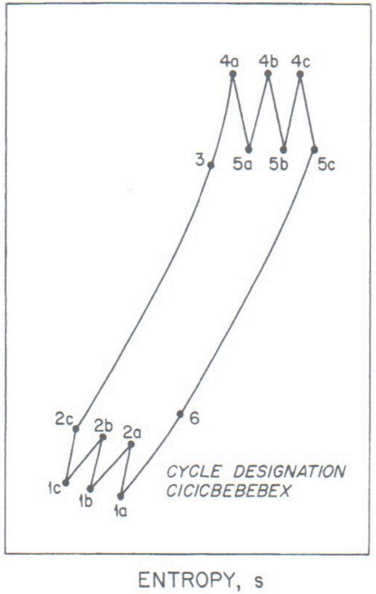
Single Intercooled



Single Intercooled & Reheated

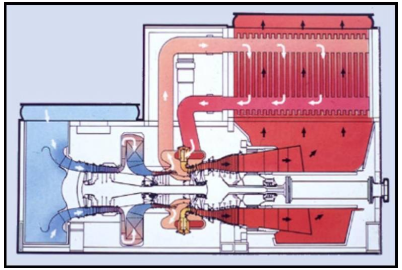


Multiple Intercooled & Reheated

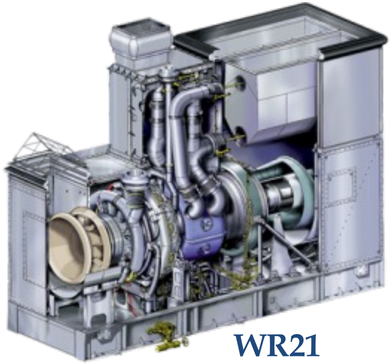


Intercooled & Recuperated Marine Engine in Service

The world's most efficient marine gas turbine

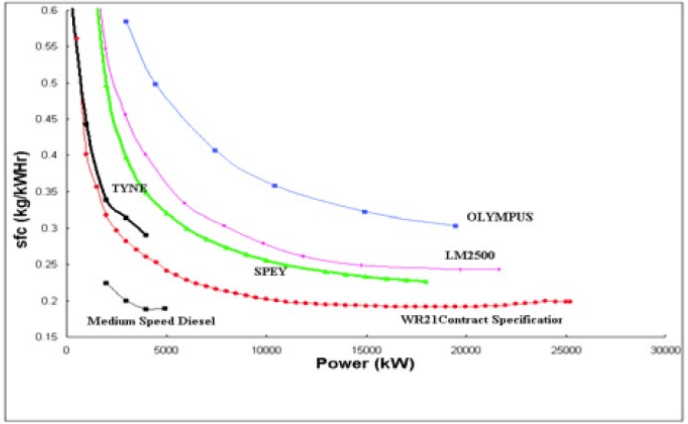


Type 45 (Royal Navy, UK)



WR21

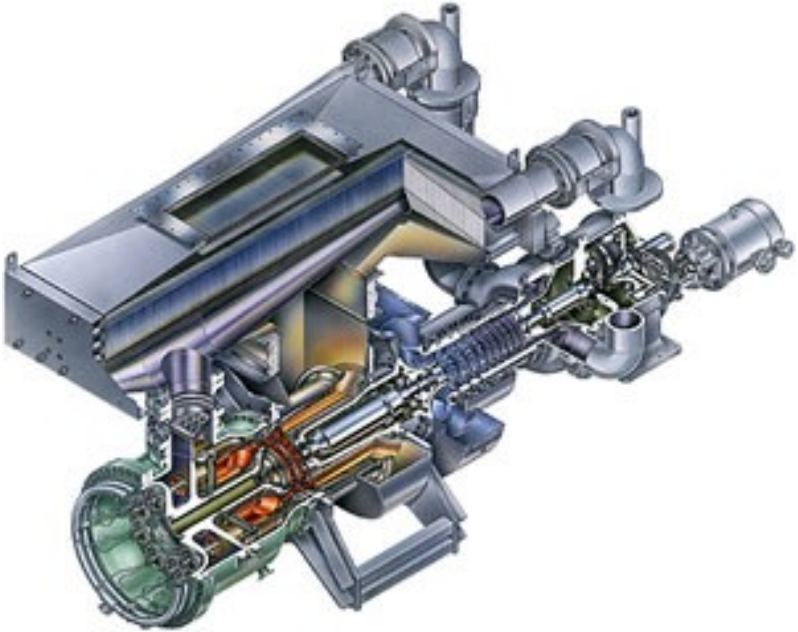
Courtesy of Rolls-Royce

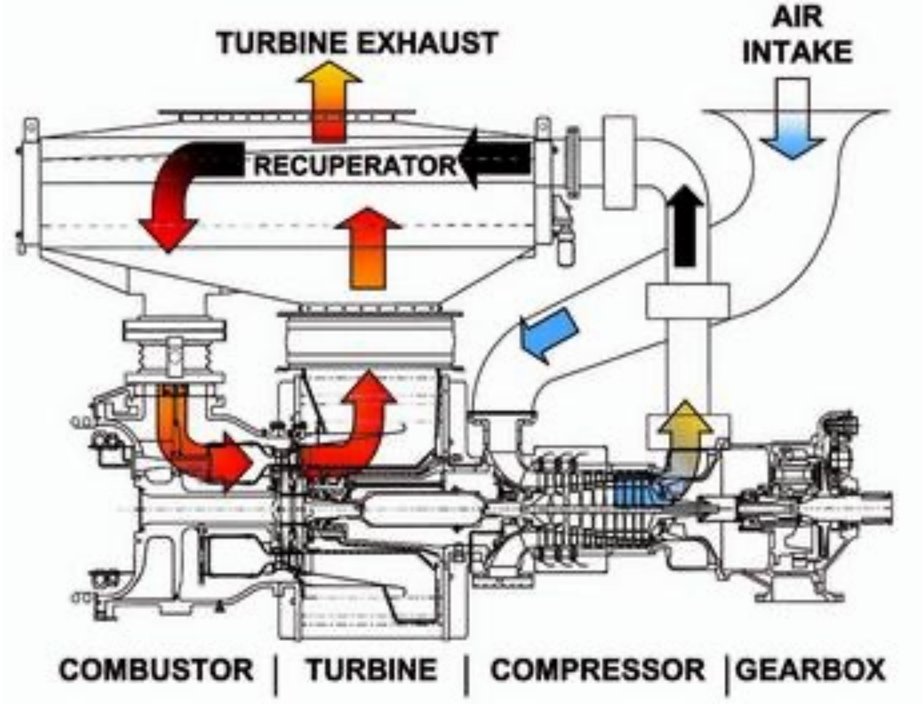


IGTC2003 Tokyo OS-203

Recuperated Gas Turbine Engines

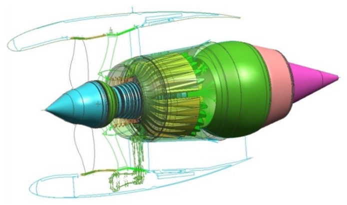
Solar Mercury 50

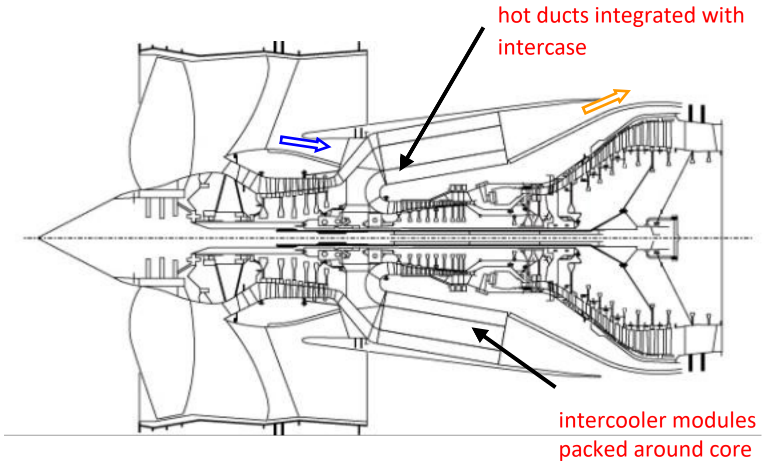




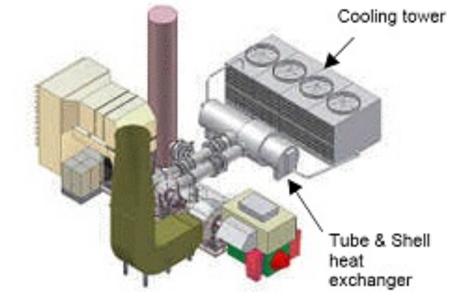
Courtesy of Solar Turbine Incorporated

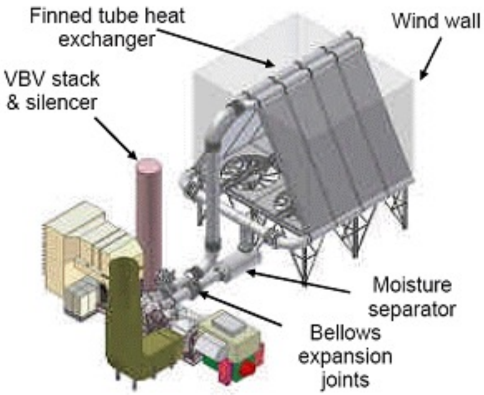
Intercooled Gas Turbine Engines





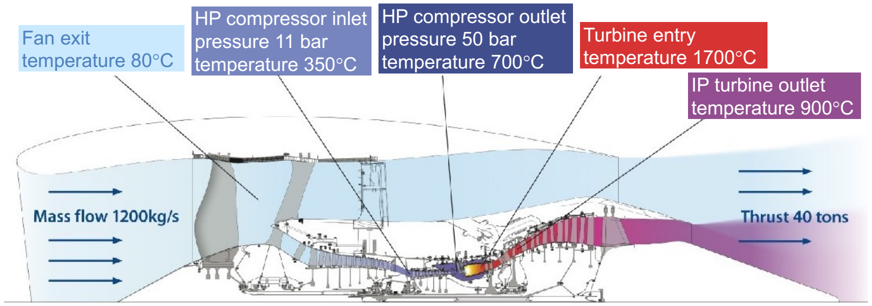
- Benefits are up to 4% SFC improvement, cooler cooling air, reduced NO_x emissions (up to 16%) and lower maintenance costs
- Light weight, low pressure ratio heat exchangers critical to concept

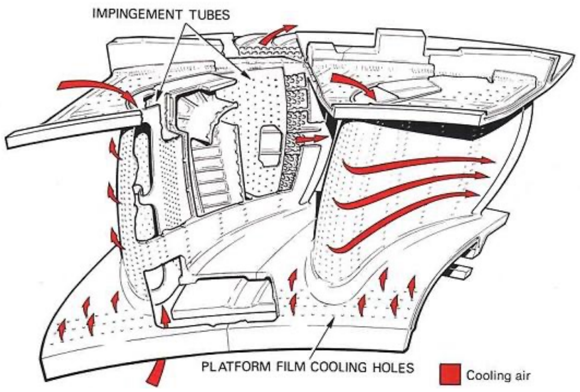


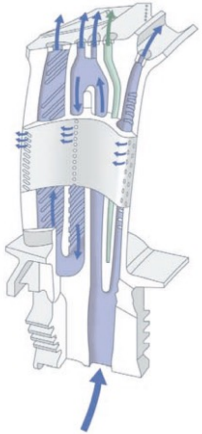


LM100 (GE)

Turbine Cooling of Gas Turbine Engine







Courtesy of Rolls-Royce

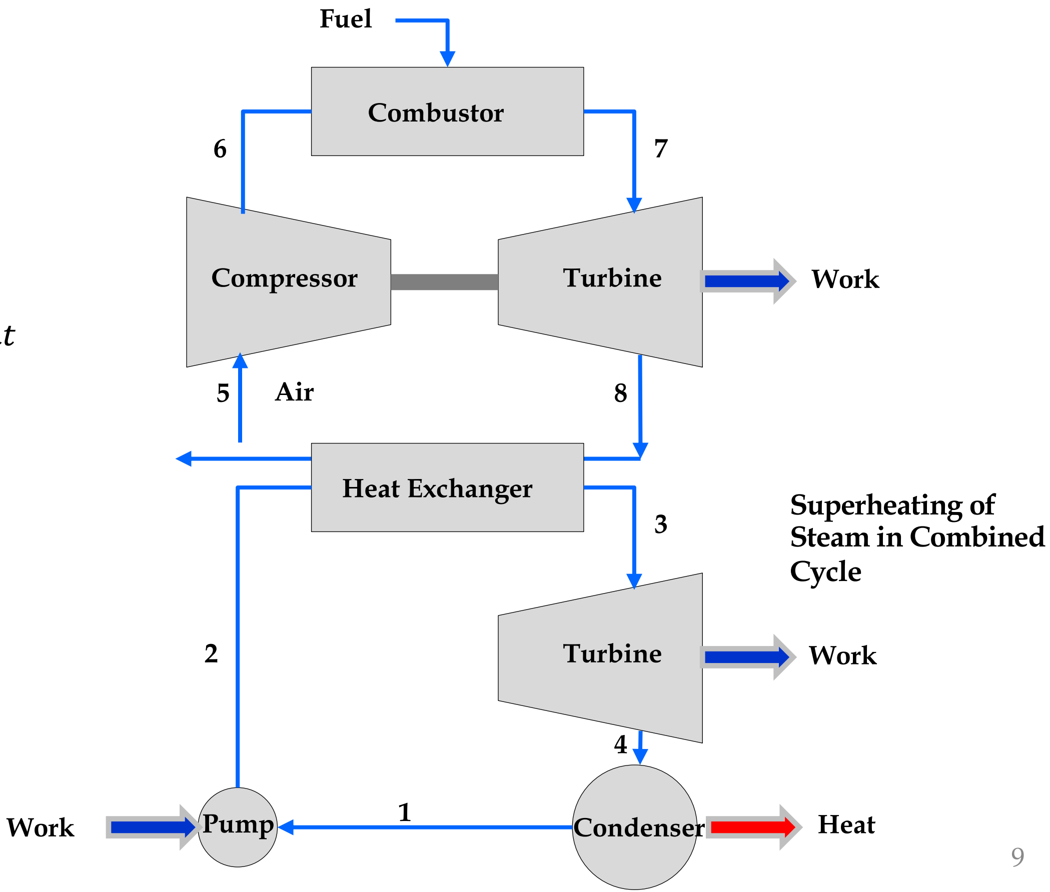
Advanced Combined Cycles for Max Efficiency

$$E_{in} = E_{out}$$

$$m_g h_{g,in} + m_s h_{s,in} = m_g h_{g,out} + m_s h_{s,out}$$

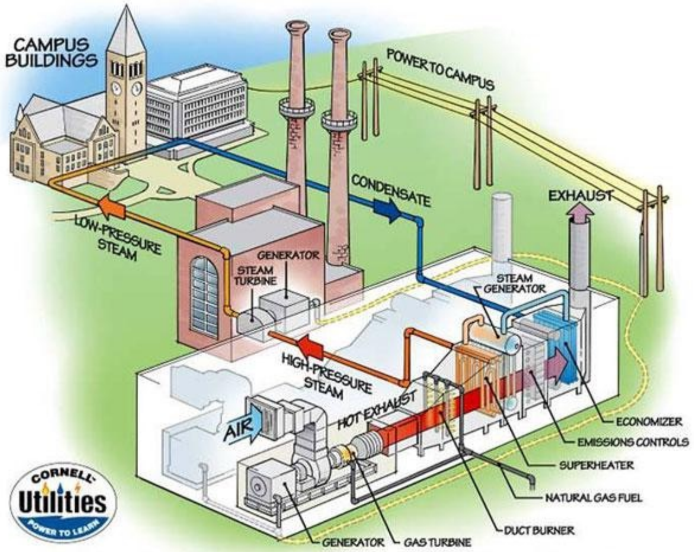
$$\frac{m_{steam}}{m_{gas}} = \frac{h_{g,in} - h_{g,out}}{h_{s,out} - h_{s,in}} = y$$

$$w_{net} = w_{gas,net} + y \times w_{steam,net}$$



Advanced Combustion Cycles for Max Efficiency

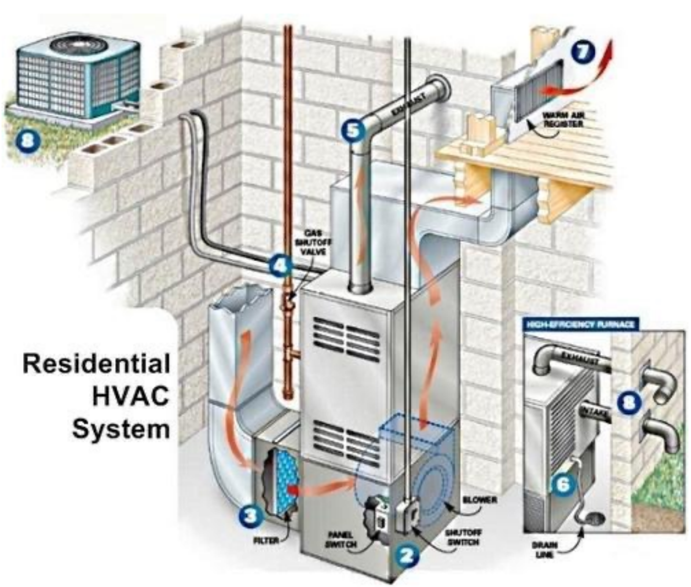
Cogeneration & Combined Heat & Power



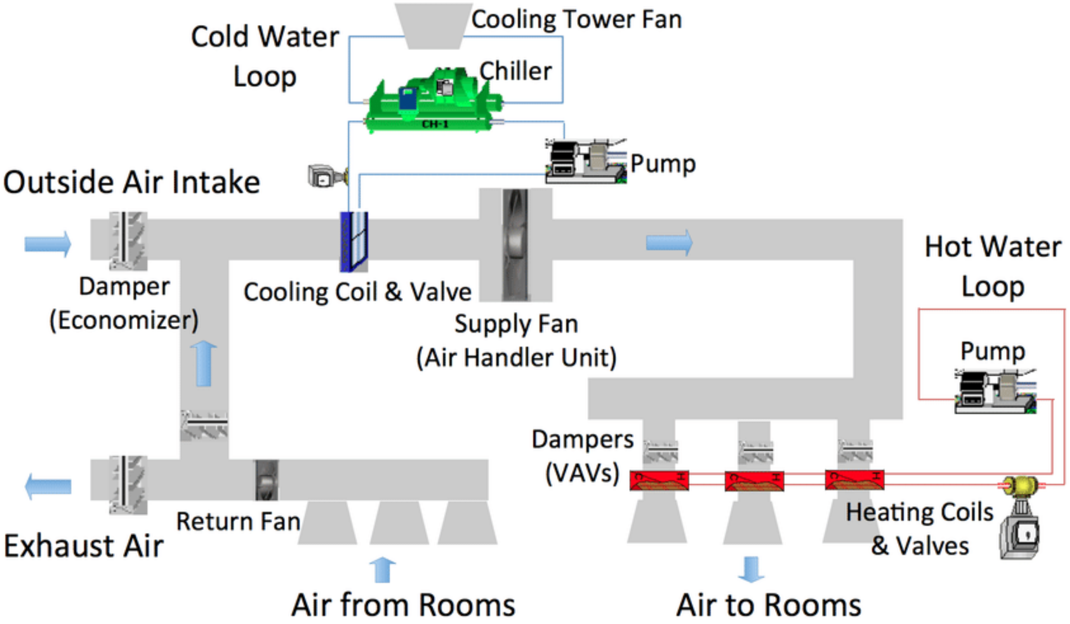
Combustion Turbine with Heat Recovery Steam Generator

Heat, Air, Ventilation and Cooling (HAVC)

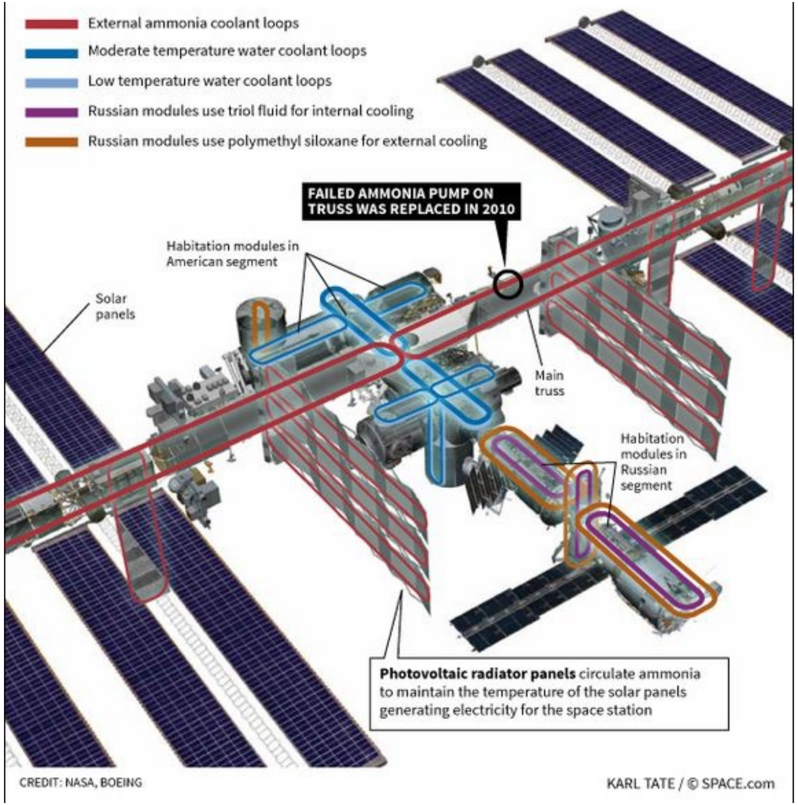
Residential

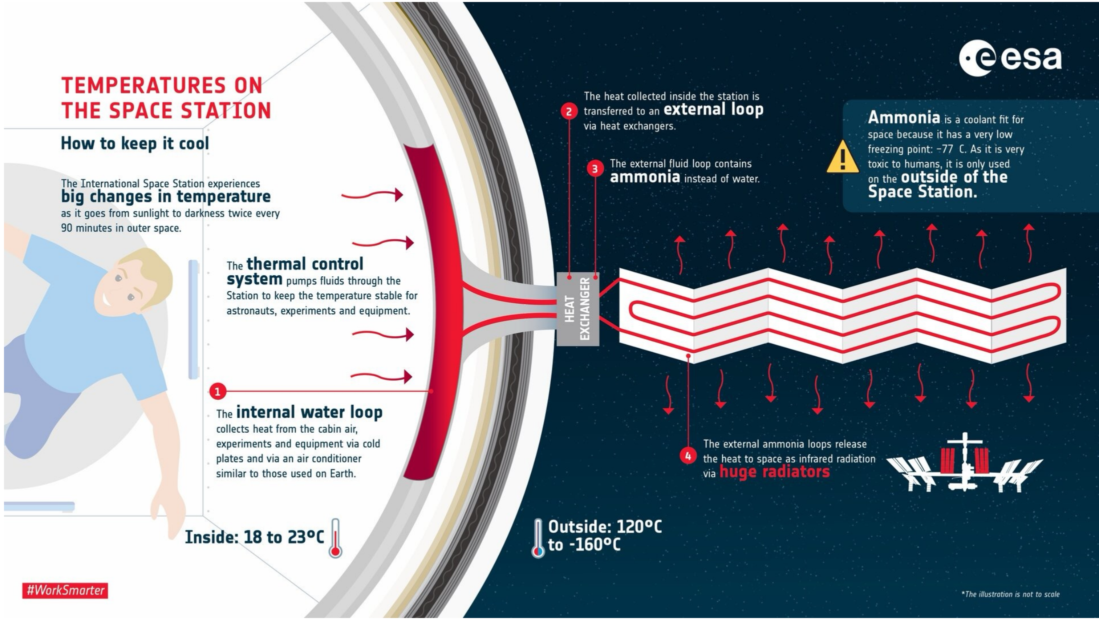


Commercial

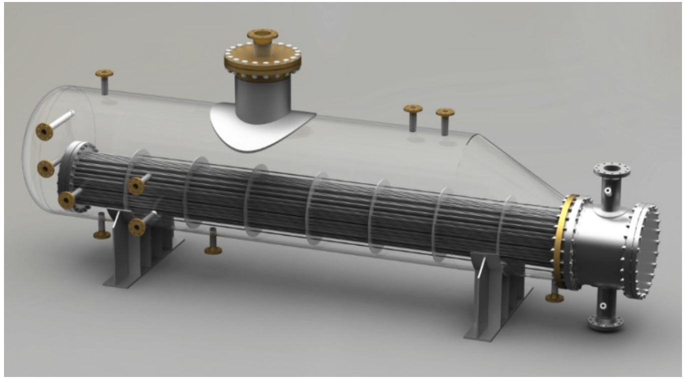


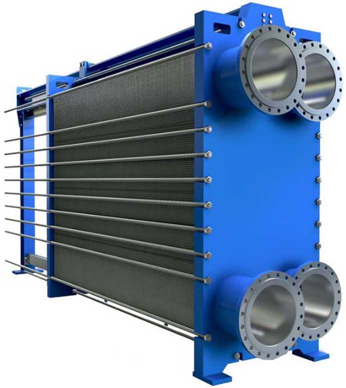
Space Station Cooling System

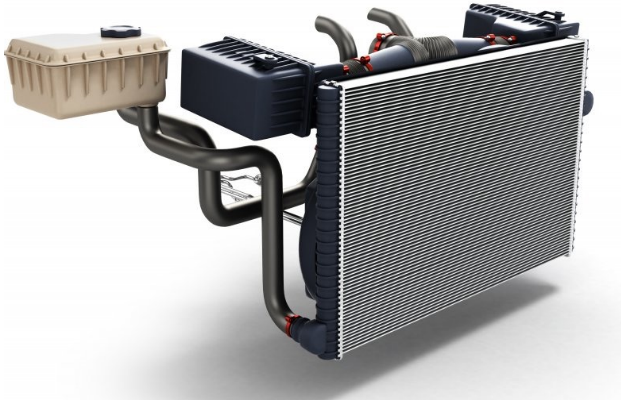


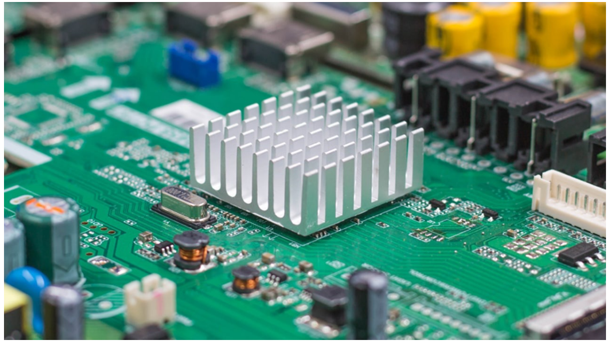


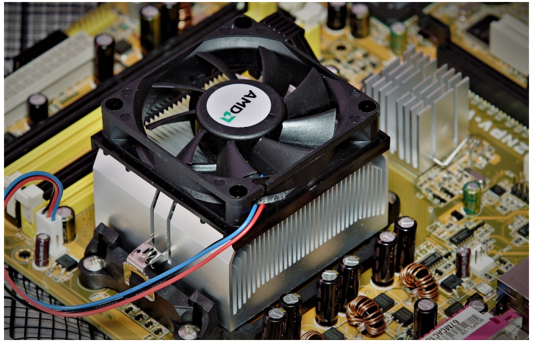
Heat Exchangers











How to enhance heat transfer?

Heat Exchanger Types

FIGURE 11.1 Concentric tube heat exchangers. (*a*) Parallel flow. (*b*) Counterflow.

Heat Exchanger Types

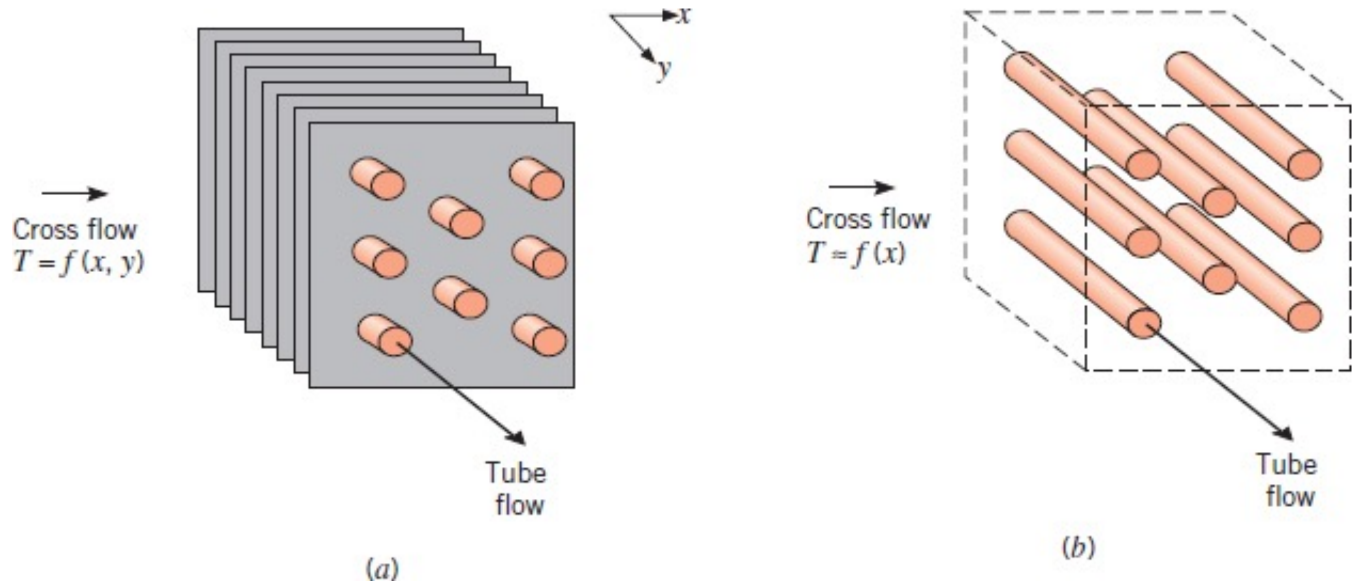


FIGURE 11.2 Cross-flow heat exchangers. (*a*) Finned with both fluids unmixed. (*b*) Unfinned with one fluid mixed and the other unmixed

Heat Exchanger Types

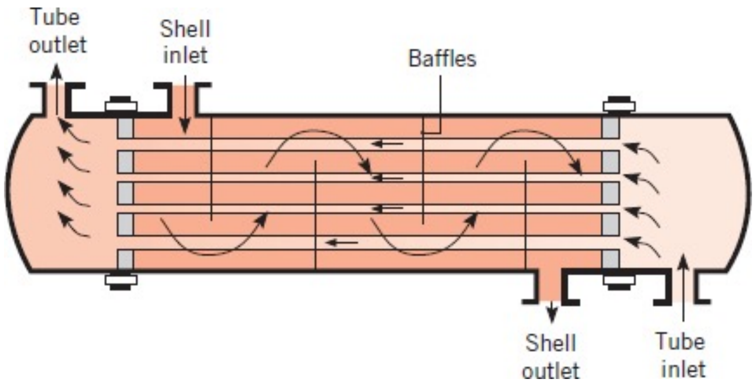


FIGURE 11.3 Shell-and-tube heat exchanger with one shell pass and one tube pass (cross-counterflow mode of operation)

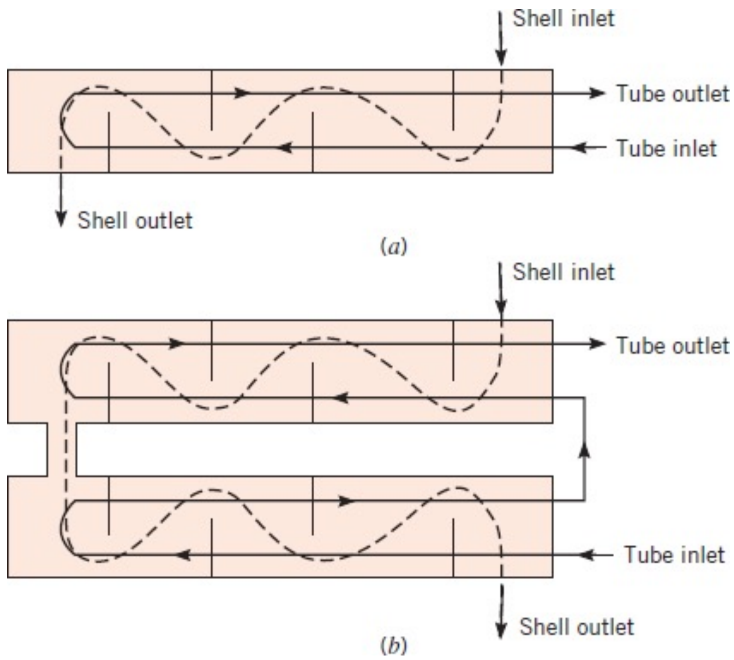
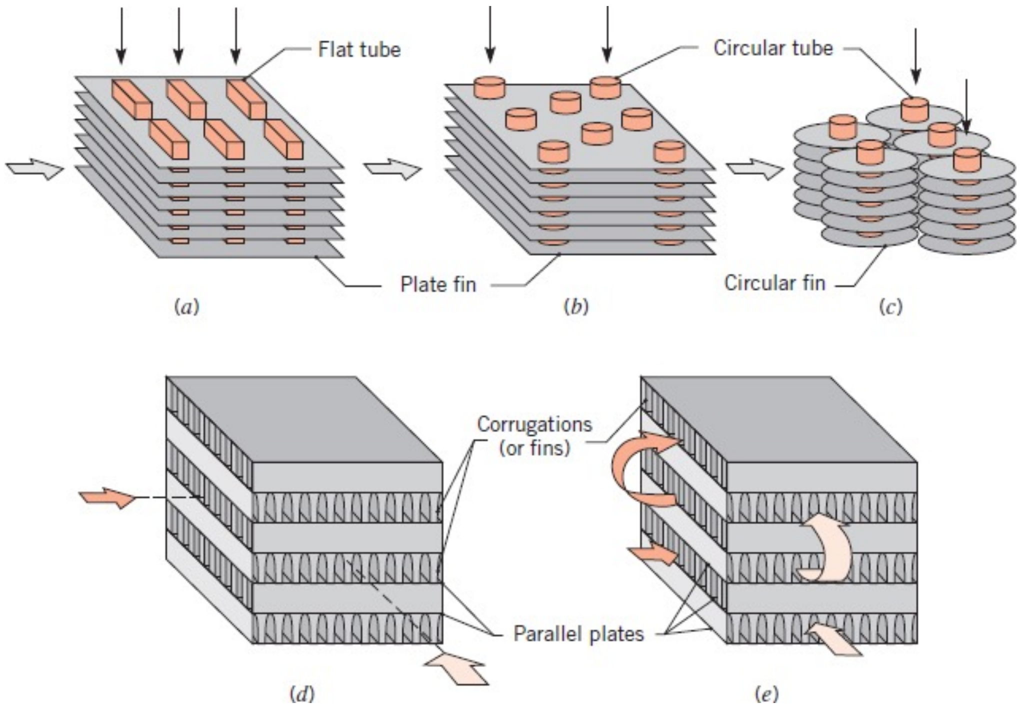


FIGURE 11.4 Shell-and-tube heat exchangers. (*a*) One shell pass and two tube passes. (*b*) Two shell passes and four tube passes.

Heat Exchanger Types



FIFURE 11.5 Compact heat exchanger cores. (*a*) Fin–tube (flat tubes, continuous plate fins). (*b*) Fin–tube (circular tubes, continuous plate fins). (*c*) Fin–tube (circular tubes, circular fins). (*d*) Plate–fin (single pass). (*e*) Plate–fin (multipass).

Modeling Approach

Log Mean Temperature Difference (LMTD) Approach

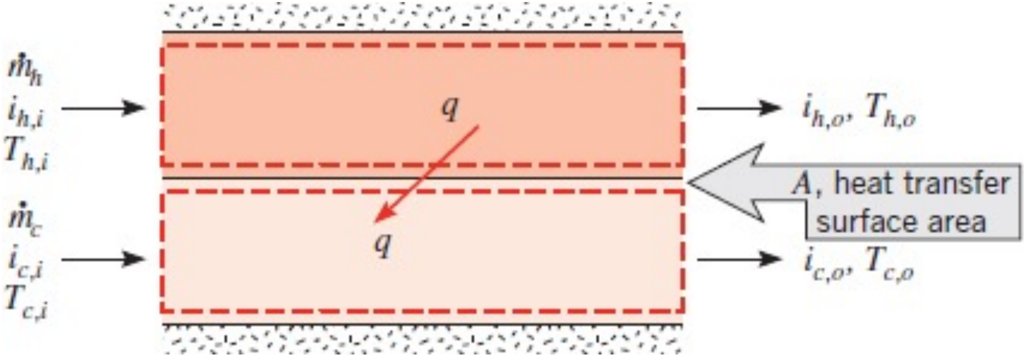


FIGURE 11.6 Overall energy balances for the hot and cold fluids of a two-fluid heat exchanger.

$$q = \dot{m}_h C_{p,h}\left(T_{h,i} - T_{h,o}\right)$$

$$q = \dot{m}_c C_{p,c}\left(T_{c,i} - T_{c,o}\right)$$

An extension of Newton's law of cooling

$$q = UA\Delta T_m \quad \text{and} \quad \frac{\Delta T}{q} = \frac{1}{UA} = R_{tot}$$

where,

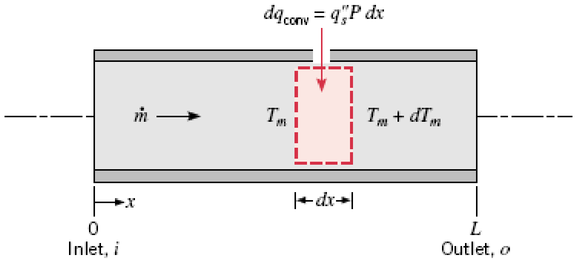
$$U = \frac{1}{R_{tot}A} \quad \text{and} \quad \Delta T \equiv T_h - T_c$$

Determination of the Mean Temperature

Chapter 8

- Determination of $T_m(x)$ is an essential feature of an internal flow analysis.

  Determination begins with an energy balance for a differential control volume.



$$dq_{\text{conv}} = \dot{m} c_p \left[\left(T_m + dT_m \right) - T_m \right] = \dot{m} c_p dT_m$$

Integrating from the tube inlet to outlet,

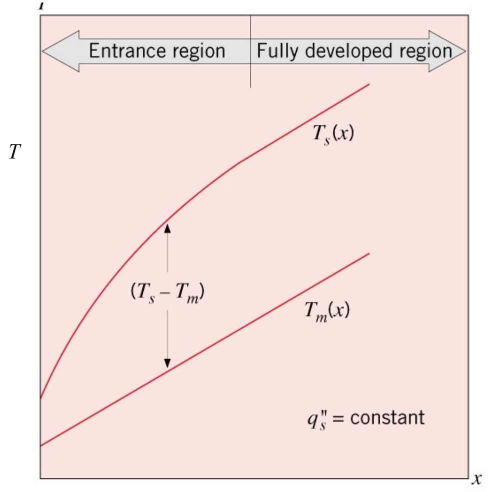
$$q_{\text{conv}} = \dot{m}\, c_p \left(T_{m,o} - T_{m,i} \right) \qquad (1)$$

Determination of the Mean Temperature

Chapter 8

A differential equation from which $T_m(x)$ may be determined is obtained by substituting for $dq_{\text{conv}} = q_s''(P\,dx) = h(T_s - T_m)P\,dx$.

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}\,c_p} = \frac{P}{\dot{m}\,c_p} h(T_s - T_m) \qquad (2)$$



- Special Case: Uniform Surface Heat Flux

Since total heat rate: $q_{\text{conv}} = q_s'' \, PL$

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}\,c_p} \neq f(x)$$

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m}\,c_p} x$$

Determination of the Mean Temperature

Chapter 8

- Special Case: Uniform Surface Temperature

From Eq. (2), with $\Delta T \equiv T_s - T_m$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\,\Delta T$$

Integrating from x=0 to any downstream location,

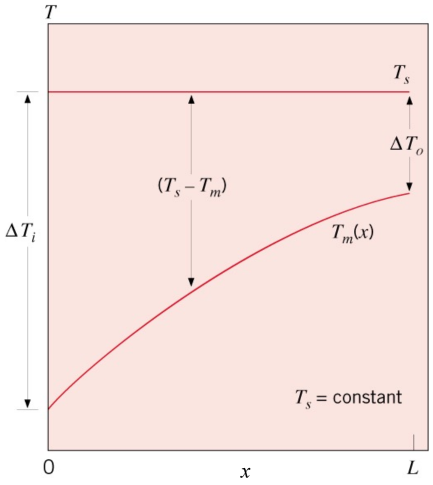
$$\ln\left[\frac{T_s - T_{m,x}}{T_s - T_{m,i}}\right] = -\left[\frac{Px}{\dot{m}c_p}\bar{h}\right] \Rightarrow \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}_x\right)$$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x\,dx$$

Overall Conditions:

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right) = \exp\left(-\frac{\bar{h}A_s}{\dot{m}c_p}\right) \Rightarrow q_{conv} = \bar{h}A_s\,\Delta T_{\ell m}$$

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)} \quad (3)$$



Determination of the Mean Temperature

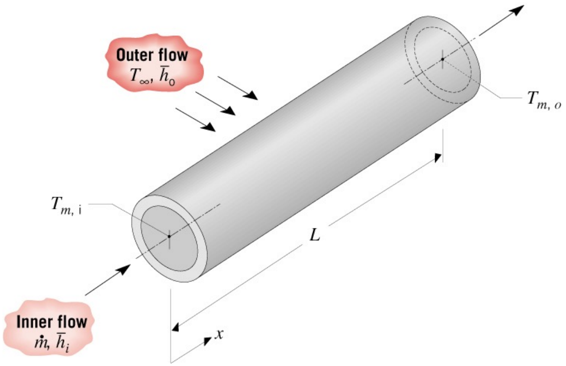
Chapter 8

- Special Case: Uniform *External* Fluid Temperature

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\bar{U} A_s}{\dot{m} c_p}\right) = \exp\left(-\frac{1}{\dot{m} c_p R_{\text{tot}}}\right)$$

$$q = \bar{U} A_s \, \Delta T_{\ell m} = \frac{\Delta T_{\ell m}}{R_{\text{tot}}}$$

$\Delta T_{\ell m} \rightarrow$ Eq. (3) with T_s replaced by T_∞.



Note: Replacement of T_∞ by $T_{s,o}$ if outer surface temperature is uniform.

Overall Heat Transfer Coefficient

For a wall separating two fluid streams, the overall heat transfer coefficient may be expressed as

$$\frac{1}{UA} = \left[\frac{1}{U_c A_c} = \frac{1}{U_h A_h}\right] = \frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h}$$

During normal heat exchanger operation, surfaces are often subject to fouling by fluid impurities, rust formation, or other reactions between the fluid and the wall material. The subsequent deposition of a film or scale on the surface can greatly increase the resistance to heat transfer between the fluids.

This effect can be treated by introducing an additional thermal resistance in termed the *fouling factor*, R_f. Its value depends on the operating temperature, fluid velocity, and length of service of the heat exchanger

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,c}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

<u>From Chapter 3</u>

where, $\eta_o = 1 - \frac{A_f}{A}(1 - \eta_f)$

Note: η_o: *overall surface efficiency* of a finned surface
A_f : the entire fin surface area
η_f : the efficiency of a single fin.

Fins with uniform cross sectional area

Chapter 3

Table 3.4 Temperature distribution and heat rates for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
A	Convection: $h\theta(L) = -kd\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.77)
B	Adiabatic: $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M \tanh mL$	(3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)
$\theta \equiv T - T_\infty$ $\theta_b = \theta(0) = T_b - T_\infty$	$m^2 \equiv hP/kA_c$ $M \equiv \sqrt{hPkA_c}\theta_b$	A table of hyperbolic functions is given in Appendix B.1.			

Overall Heat Transfer Coefficient

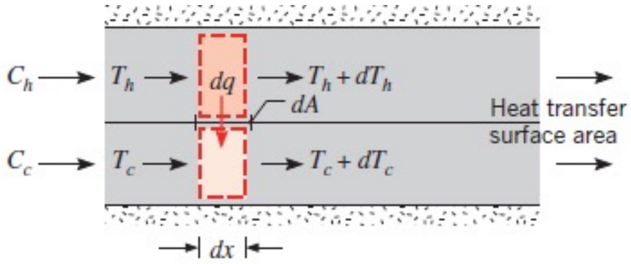
For finned surface
$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,c}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

For un-finned surface
$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$= \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{ln(D_o/D_i)}{2\pi kL} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

where subscripts *i* and *o* refer to inner and outer tube surfaces ($A_i = \pi D_i L$, $A_o = \pi D_o L$), which may be exposed to either the hot or the cold fluid.

The Parallel-Flow Heat Exchanger



$$dq = -\dot{m}_h c_{p,h}\, dT_h \equiv -C_h\, dT_h$$

$$dq = \dot{m}_c c_{p,c}\, dT_c \equiv C_c\, dT_c$$

where, C_h and C_c are the hot and cold fluid *heat capacity rates*, respectively.

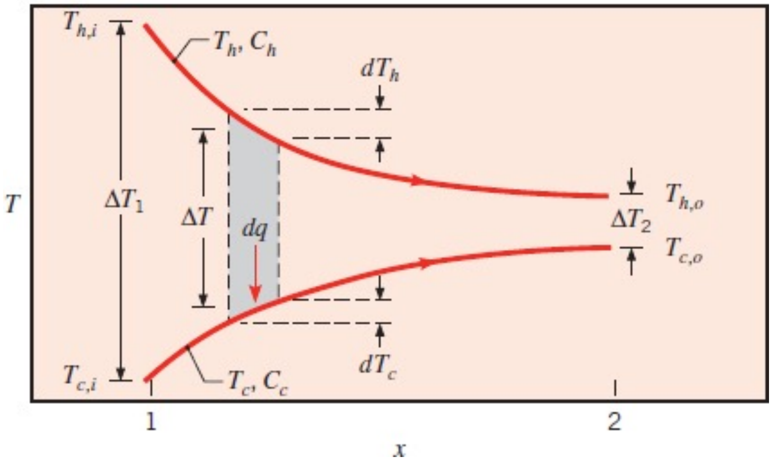


FIGURE 11.7 Temperature distributions for a parallel-flow heat exchanger.

The rate of heat transfer across the surface area dA may also be expressed as $dq = U \Delta T\, dA$

$$\Delta T \equiv T_h - T_c \longrightarrow d(\Delta T) = dT_h - dT_c$$

$$d(\Delta T) = -dq \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U \left(\frac{1}{C_h} + \frac{1}{C_c} \right) \int_1^2 dA$$

$$\ln \left(\frac{\Delta T_2}{\Delta T_1} \right) = -UA \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

The Parallel-Flow Heat Exchanger

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \longrightarrow \ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_{h,i} - T_{h,o}}{q} + \frac{T_{c,o} - T_{c,i}}{q}\right)$$

$$= -\frac{UA}{q}[(T_{h,i} - T_{c,i}) - (T_{h,o} - T_{c,o})]$$

$$dq = -\dot{m}_h c_{p,h}\, dT_h \equiv -C_h\, dT_h$$

$$dq = \dot{m}_c c_{p,c}\, dT_c \equiv C_c\, dT_c$$

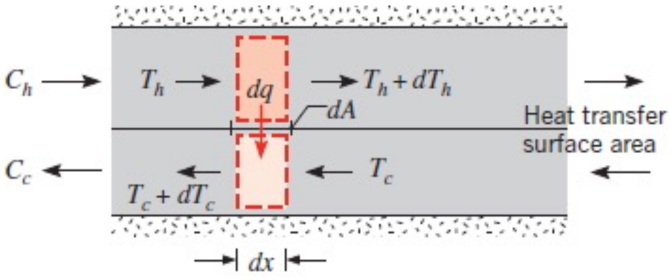
For the parallel-flow heat exchanger,
$\Delta T_1 = (T_{h,i} - T_{c,i})$ and $\Delta T_2 = (T_{h,o} - T_{c,o})$ $\longrightarrow$

$$\left[\begin{array}{l} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o} \end{array}\right]$$

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

$$q = UA\Delta T_{lm}$$

The Counter-Flow Heat Exchanger



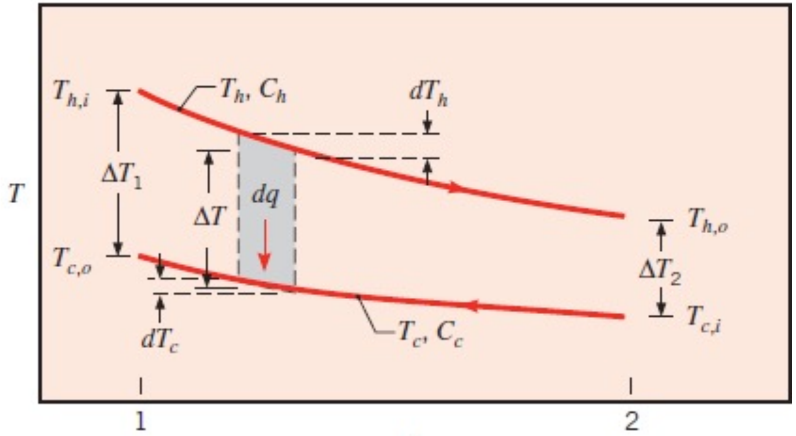


FIGURE 11.8 Temperature distributions for a counter-flow heat exchanger.

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

For counter-flow heat exchanger

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - \underline{T_{c,o}} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - \underline{T_{c,i}} \end{bmatrix}$$

For parallel-flow heat exchanger

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - \underline{T_{c,i}} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - \underline{T_{c,o}} \end{bmatrix}$$

The Counter-Flow Heat Exchanger

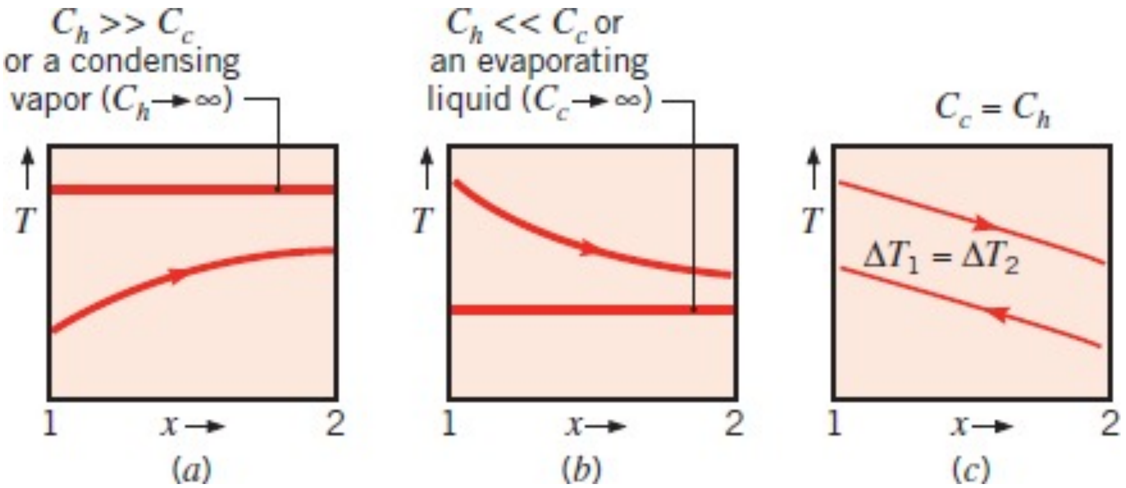


FIGURE 11.9 Special heat exchanger conditions. (*a*) $C_h \gg C_c$ or a condensing vapor. (*b*) An evaporating liquid or $C_h \ll C_c$. (*c*) A counter-flow heat exchanger with equivalent fluid heat capacities ($C_h = C_c$).

The Effectiveness–NTU Method

Use the log mean temperature difference (LMTD) method of heat exchanger analysis when the fluid inlet temperatures are known and the outlet temperatures are specified or readily determined from the energy balance expressions. The value of ΔT_{lm} for the exchanger may then be determined.

However, if only the inlet temperatures are known, use of the LMTD method requires a cumbersome iterative procedure. It is therefore preferable to employ an alternative approach termed the effectiveness–NTU (or NTU) method.

The Effectiveness–NTU Method

$$\text{Effectiveness}, \varepsilon = \frac{\text{actual heat transfer}}{\text{max. possible heat transfer}} = \frac{q_{actual}}{q_{max}}$$

The maximum possible heat transfer rate, q_{max}, could be achieved in a counter-flow heat exchanger of infinite length. In such an exchanger, the maximum possible temperature difference is $T_{h,i} - T_{c,i}$.

If $C_c < C_h$, the cold fluid would then experience the larger temperature change, and since $L \to \infty$, it would be heated to the inlet temperature of the hot fluid ($T_{c,o} = T_{h,i}$).

$$C_c < C_h : \qquad q_{\max} = C_c \left(T_{h,i} - T_{c,i}\right)$$

Similarly, if $C_h < C_c$, the hot fluid would experience the larger temperature change and would be cooled to the inlet temperature of the cold fluid ($T_{h,o} = T_{c,i}$).

$$C_h < C_c : \qquad q_{\max} = C_h \left(T_{h,i} - T_{c,i}\right)$$

The Effectiveness–NTU Method

For general expression, $q_{max} = C_{min}(T_{h,i} - T_{c,i})$

$$q_{actual} = \varepsilon C_{min}(T_{h,i} - T_{c,i})$$

$$\varepsilon = \frac{q_{actual}}{q_{max}} \longrightarrow \varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})} \quad \text{or} \quad \varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})}$$

The Effectiveness–NTU Method

To determine a specific form of the effectiveness–NTU relation, consider a parallel-flow heat exchanger for which $C_{min} = C_h$.

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})} \longrightarrow \varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$$

$$\left.\begin{aligned} q &= \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) \\ q &= \dot{m}_c c_{p,c}(T_{c,i} - T_{c,o}) \end{aligned}\right\} \qquad \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \longrightarrow \ln\left(\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}\right) = -\frac{UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)$$

$$\boxed{\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\mathrm{NTU}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}$$

The Effectiveness–NTU Method

To determine a specific form of the effectiveness–NTU relation, consider a parallel-flow heat exchanger for which $C_{min} = C_h$.

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \frac{T_{h,o} - T_{h,i} + T_{h,i} - T_{c,o}}{T_{h,i} - T_{c,i}} \longleftarrow \frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\mathrm{NTU}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]$$

$$T_{c,o} = \frac{C_{min}}{C_{max}}(T_{h,i} - T_{h,o}) + T_{c,i} \longleftarrow \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$$

$$= \frac{(T_{h,o} - T_{h,i}) + (T_{h,i} - T_{c,i}) - (C_{\min}/C_{\max})(T_{h,i} - T_{h,o})}{T_{h,i} - T_{c,i}} \longleftarrow \varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$$

$$= -\varepsilon + 1 - \left(\frac{C_{\min}}{C_{\max}}\right)\varepsilon = 1 - \varepsilon\left(1 + \frac{C_{\min}}{C_{\max}}\right)$$

$$\varepsilon = \frac{1 - \exp\{-\mathrm{NTU}[1 + (C_{\min}/C_{\max})]\}}{1 + (C_{\min}/C_{\max})}$$

The Effectiveness–NTU Method

$$\varepsilon = \frac{1 - \exp\{-\mathrm{NTU}[1 + (C_{\min}/C_{\max})]\}}{1 + (C_{\min}/C_{\max})}$$

$$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right)$$ where, $C_{\min}/C_{\max}$ is equal to C_c/C_h or C_h/C_c

and number of transfer unit, $NTU \equiv \frac{UA}{C_{min}}$

TABLE 11.3 Heat Exchanger Effectiveness Relations

Flow Arrangement	**Relation**	
Parallel flow	$$\varepsilon = \frac{1 - \exp[-\mathrm{NTU}(1 + C_r)]}{1 + C_r}$$	(11.28a)
Counterflow	$$\varepsilon = \frac{1 - \exp[-\mathrm{NTU}(1 - C_r)]}{1 - C_r \exp[-\mathrm{NTU}(1 - C_r)]} \quad (C_r < 1)$$ $$\varepsilon = \frac{\mathrm{NTU}}{1 + \mathrm{NTU}} \quad (C_r = 1)$$	(11.29a)
Shell-and-tube		
One shell pass (2, 4, ... tube passes)	$$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$$	(11.30a)
n shell passes ($2n$, $4n$, ... tube passes)	$$\varepsilon = \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$$	(11.31a)
Cross-flow (single pass)		
Both fluids unmixed	$$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\left\{\exp[-C_r(\mathrm{NTU})^{0.78}] - 1\right\}\right]$$	(11.32)
C_{max} (mixed), C_{min} (unmixed)	$$\varepsilon = \left(\frac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\})$$	(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\mathrm{NTU})]\})$$	(11.34a)
All exchangers (C_r = 0)	$$\varepsilon = 1 - \exp(-\mathrm{NTU})$$	(11.35a)

TABLE 11.4 Heat exchanger NTU relations

$$\mathrm{NTU} = f\left(\varepsilon, \frac{C_{\min}}{C_{\max}}\right)$$

Flow Arrangement	Relation	
Parallel flow	$\mathrm{NTU} = -\frac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$	(11.28b)
Counterflow	$\mathrm{NTU} = \frac{1}{C_r - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$ $\mathrm{NTU} = \frac{\varepsilon}{1-\varepsilon} \quad (C_r = 1)$	(11.29b)
Shell-and-tube		
One shell pass (2, 4, ... tube passes)	$(\mathrm{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\frac{E-1}{E+1}\right)$	(11.30b)
	$E = \frac{2/\varepsilon_1 - (1+C_r)}{(1+C_r^2)^{1/2}}$	(11.30c)
n shell passes ($2n$, $4n$, ... tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \frac{F-1}{F-C_r} \quad F = \left(\frac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$	(11.31b, c, d)
Cross-flow (single pass)		
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\frac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$	(11.33b)
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\mathrm{NTU} = -\left(\frac{1}{C_r}\right)\ln[C_r \ln(1-\varepsilon) + 1]$	(11.34b)
All exchangers (C_r = 0)	$\mathrm{NTU} = -\ln(1-\varepsilon)$	(11.35b)

Effectiveness - NTU

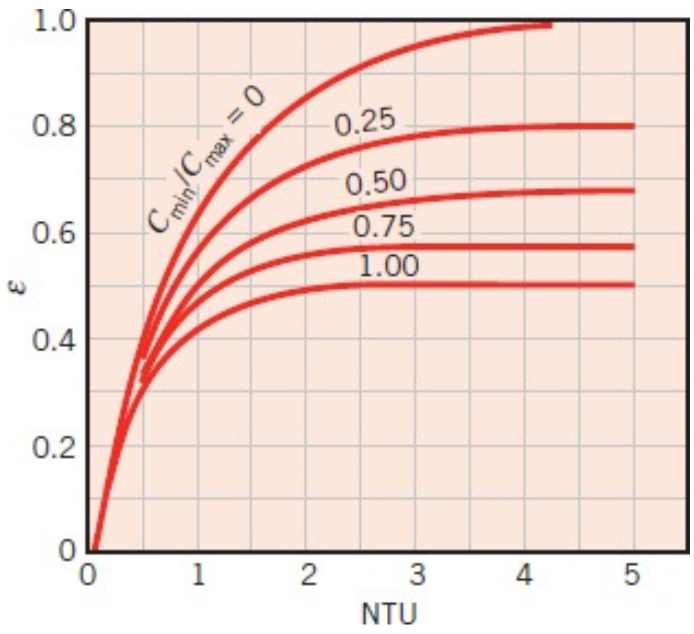


FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger

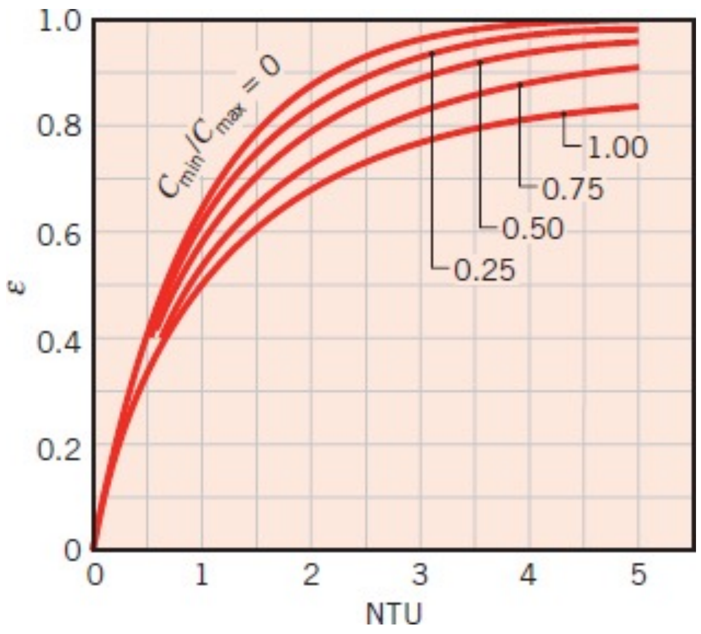


FIGURE 11.11 Effectiveness of a counter-flow heat exchanger

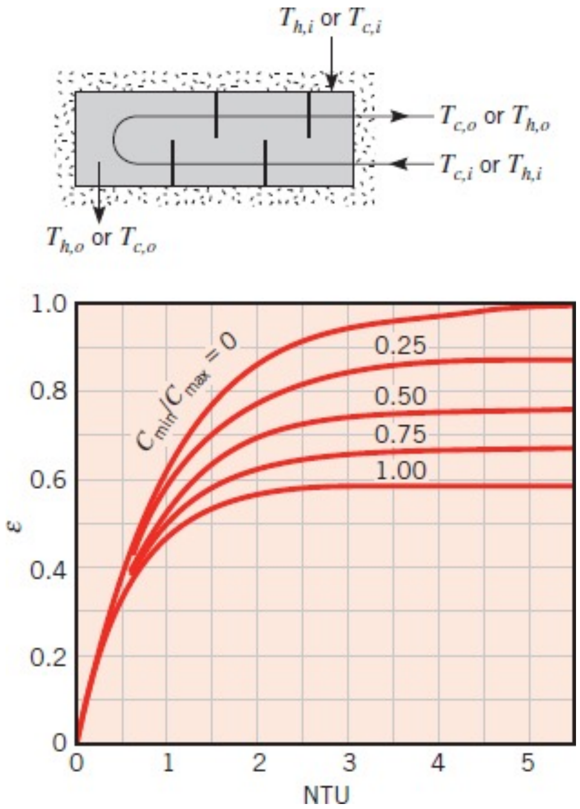


FIGURE 11.12 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes)

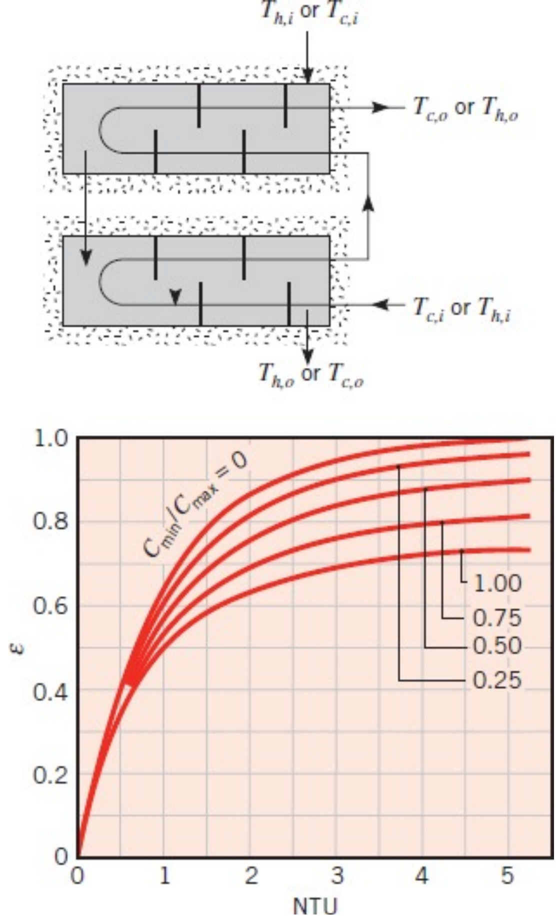


FIGURE 11.13 Effectiveness of a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes)

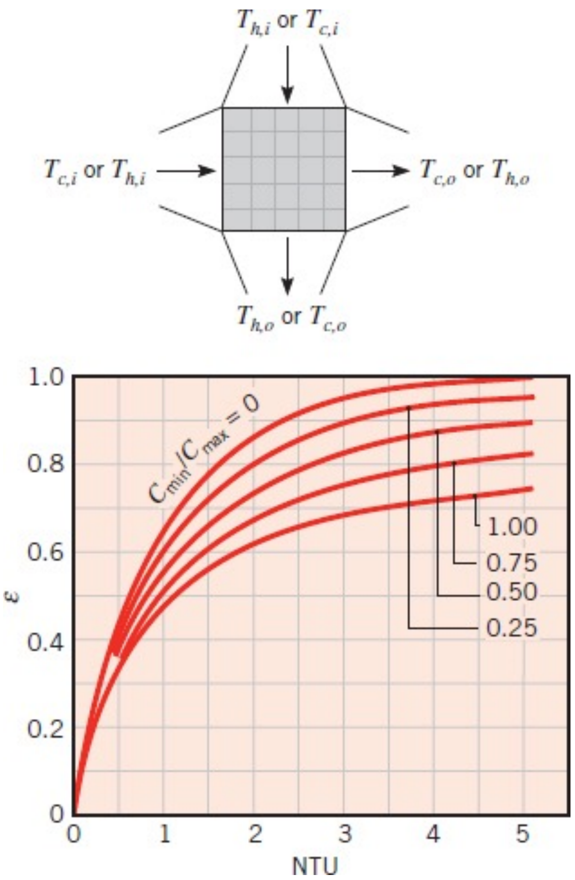


FIGURE 11.14 Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed

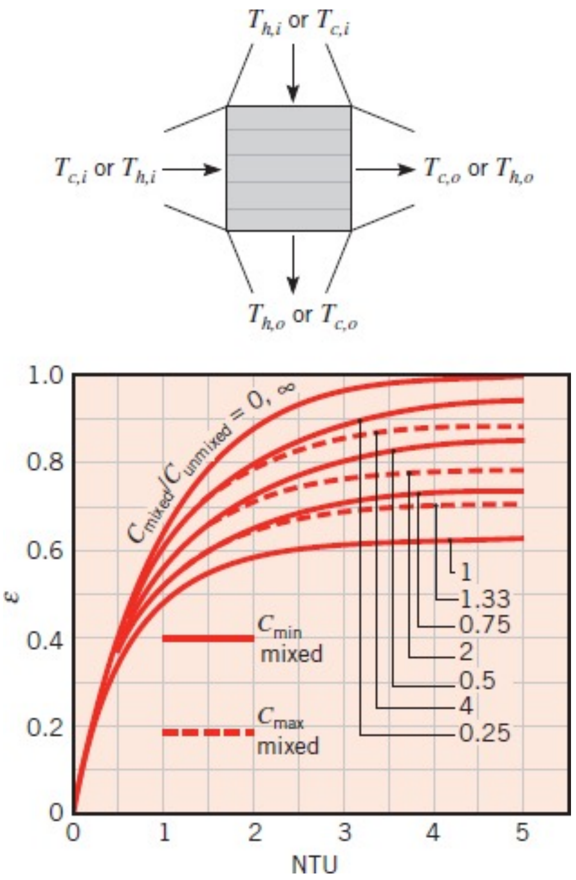


FIGURE 11.15 Effectiveness of a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed