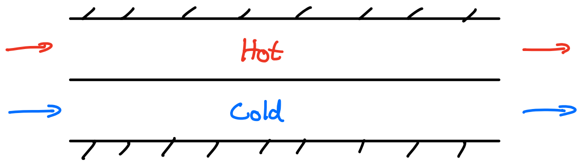
12C - 2ND CLASS

PARELLEL FLOW HX



$q = UA\Delta T_{lm}$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

$$UA = \frac{1}{R_{total}} \qquad q = UA\Delta T = \frac{\Delta T}{R_{total}}$$

Nondimensionalize

$$\frac{q}{q_{max}} = \varepsilon = \text{effectiveness}$$

$$\frac{UA}{C_{min}} = NTU \qquad \frac{C_{min}}{C_{max}} = C_r = \text{ratio of heat capacity rates}$$

Effectiveness - NTU method

q_{max} is when one fluid undergoes maximum possible temp change: $T_{h,i} - T_{c,i}$

This can only occur for the fluid with C_{min}

$$q_{max} = C_{min}(T_{h,i} - T_{c,i})$$

$$C_{min} = \min\{C_h, C_c\}$$

$C \equiv \dot{m}c_p$
$C_h = \dot{m}_h c_{p_h}$
$C_c = \dot{m}_c c_{p_c}$

Assume

$C_{min} = C_h$

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$$

$0 \leq \varepsilon \leq 1$

$$C_r = \frac{C_{min}}{C_{max}} = \frac{\dot{m}_h c_{p_h}}{\dot{m}_c c_{p_c}} = \frac{\cancel{q}}{T_{h,i} - T_{h,o}} \frac{T_{c,o} - T_{c,i}}{\cancel{q}} \rightarrow \boxed{C_r = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}}$$

Parallel Flow HX

Recall,

$$\ln(\Delta T_2/\Delta T_1) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\ln\left(\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}\right) = -UA\left(\frac{1}{C_{min}} + \frac{1}{C_{max}}\right) = \frac{-UA}{C_{min}}(1 + C_r)$$

$$\frac{T_{h,o}-T_{c,o}}{T_{h,i}-T_{c,i}} = e^{-NTU(1+C_r)}$$

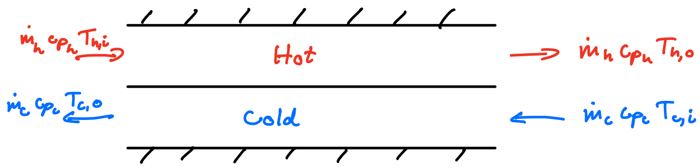
After some algebra:

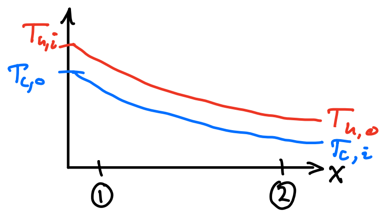
$$\boxed{\varepsilon = \frac{1-e^{-NTU(1+C_r)}}{1+C_r}}$$

← ε-NTU relationship for a parallel-flow HX

Note: The result is the same if $C_{min} = C_c$

COUNTER-FLOW HX





$T_{c,o}$ can be greater than $T_{h,o}$