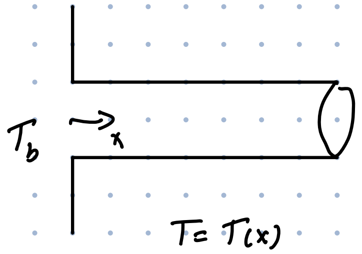
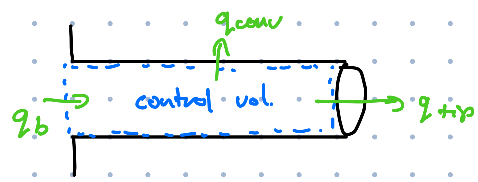
4E FINS CONTINUED



$$\theta(x) = \theta_b \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$q_f = ?$



$$q_b = q_{conv} + q_{tip} = \underset{\text{fin}}{q_f}$$

$$q_f = q_b = -k A_{c,b} \frac{dT}{dx}\Big|_{x=0}$$

cross-sectional area at the base

$$\cancel{\dot{E}_{st}} = \dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g}$$

stored? generated

$$\dot{E}_{in} = \dot{E}_{out}$$

$\theta(x) = T(x) - T_b$

Plug in our $T(x)$,

$$q_f = k A_{c,b}\, m (T_b - T_\infty) \tanh(mL)$$

hyperbolic tangent

see table 3.4 for many types of fins!

How good is a fin?

ϵ_f = Fin effectiveness

$$= \frac{q_f}{q_{\text{without a fin}}} = \frac{q_f}{h A_{c,b}(T_b - T_\infty)} = \epsilon_f$$ ← should be greater than 1

η_f = fin efficiency

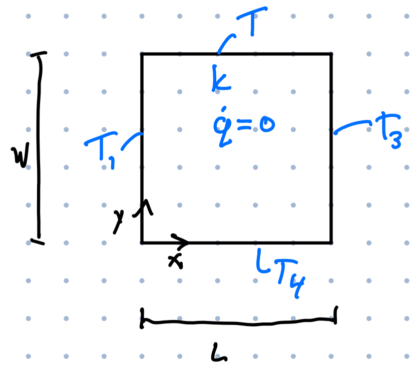
should be between 0 and 1

$$= \frac{q_f}{q_{max}} = \frac{q_f}{h A_f (T_b - T_\infty)}$$

if the entire fin were at T_b

A_f – total surface area of the fin including the tip

2D Steady Conduction



$T(x,y) = ?$

$\vec{q}(x,y) = ?$

Assumptions
- Steady
- Constant Prop

Heat Equation

$$\cancel{\frac{\partial T}{\partial t}}^{0} = \alpha \nabla^2 T \quad \cancel{\frac{\dot{q}}{\rho c_p}}^{0}$$

$$\nabla^2 T = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{0} = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

PDE
2nd order → 2 B.C. in x, 2 B.C. in y
linear
Homogeneous

B.C.
1. $T(x=0, y) = T_1$
2. $T(x, y=W) = T_2$
3. $T(x=L, y) = T_3$
4. $T(x, y=0) = T_4$

all nonhomogeneous

How to solve?

1. Mathematically
   - "Seperation of Variables"

2. Approx numerical solution
   - "Finite Difference"