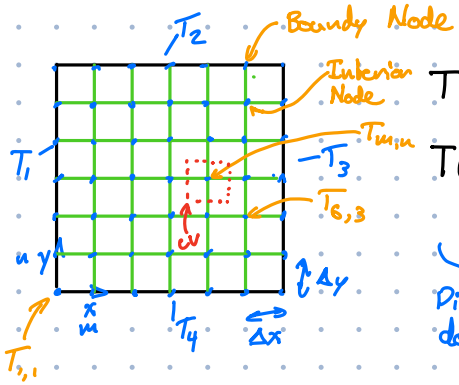
5A - FINITE DIFFERENCE

L_c - "corrected length"? - Fig. 3-18

Array of Fins



$T = T(x,y)$

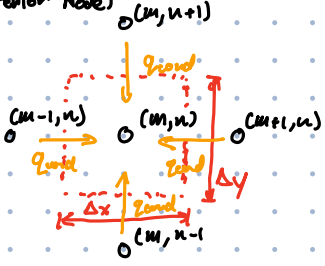
$T(x,y) \rightarrow T_{m,n}$

Discretized the domain

PDE → Many algebraic equations } easy on a computer

Conservation of Energy:

(Interior Node)



$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$

storage, generated

$\Rightarrow \dot{E}_{in} = 0$

$$q_{cond}\Big|_{m-1,n} + q_{cond}\Big|_{m+1,n} + q_{cond}\Big|_{m,n-1} + q_{cond}\Big|_{m,n+1} = 0$$

$$q_{cond}\Big|_{m-1,n} = -kA\frac{dT}{dx}$$

$A = \Delta y\,(1)$

$$\frac{dT}{dx}\Big|_{m-1,n} = ?$$

Recall

$f(x)$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{df}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$ } this becomes exact when $\Delta x \to 0$

So,

$$\frac{dT}{dx}\Big|_{m-1,n} \approx \frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

$$q_{cond}\Big|_{m-1,n} = -k\Delta y\,(1)\left(\frac{T_{m,n} - T_{m-1,n}}{\Delta x}\right)$$

$$q_{cond}\Big|_{m-1,n} = k\Delta y\,(1)\left(\frac{T_{m-1,n} - T_{m,n}}{\Delta x}\right)$$

$$q_{m+1,n} = -\left(-k\Delta y(1)\frac{dT}{dx}\Big|_{m+n,n}\right)$$ (pointing in the +ve direction)

$$= +k\Delta y(1)\left(\frac{T_{m+1,n} - T_{m,n}}{\Delta x}\right)$$

These can now be written down

$$q_{cond}\big|_{m-1,n} + q_{cond}\big|_{m,n+1} + q_{cond}\big|_{m+1,n} + q_{cond}\big|_{m,n-1} = 0$$

$$\Rightarrow + k\Delta y(1)\left(\frac{T_{m-1,n} - T_{m,n}}{\Delta x}\right) + k\Delta x(1)\left(\frac{T_{m,n+1} - T_{m,n}}{\Delta y}\right) + k\Delta y(1)\left(\frac{T_{m+1,n} + T_{m,n}}{\Delta x}\right)$$

$$+ k\Delta x(1)\left(\frac{T_{m,n-1} - T_{m,n}}{\Delta y}\right) = 0$$

IF $\Delta x = \Delta y$,

$$T_{m-1,n} + T_{m+1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = 0$$

Interior Node, Conduction Only