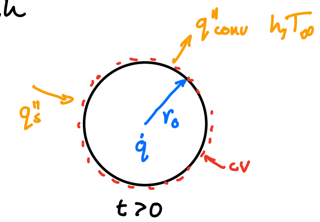
6E - TRANSIENT HEAT TRANSFER

Use a lumped cap. approach to derive an Eq. for $T(t)$

sphere - radius r_o
T_o at $t=0$
$t>0$
volumetric heat generation
$\dot{q} = aT^2$
imposed heat flux
$q'' = \left(\frac{b r_o}{3}\right) T$
convecting fluid
h, T_∞
$T_o > T_\infty$
a, b constant

1. Sketch



$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$$\underbrace{\rho c V}_{V}\frac{dT}{dt} = \underbrace{q''_s A_s}_{V} - \underbrace{hA_s(T_s - T_\infty)}_{V}$$

Vol $\frac{4}{3}\pi r^3$ SA $4\pi r^2$

$$\rho c \frac{dT}{dt} = \frac{q''_s A_s}{V} - \frac{hA_s(T_s - T_\infty)}{V}$$

$$\frac{A_s}{V} = \frac{2\pi r_o^2}{4\pi r_o^2} \cdot 3 = \frac{3}{r_o}$$

$$\rho c \frac{dT}{dt} = \frac{b r_o}{3}\frac{3}{r_o} T - \frac{3h_o}{r_o}(T - T_\infty) + aT^3$$

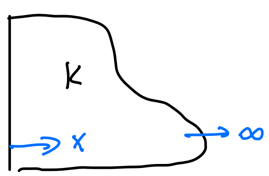
$$\boxed{\begin{aligned} &\rho c \frac{dT}{dt} = bT - \frac{3h}{r_o}(T - T_\infty) + aT^2 \\ &T(t=0) = T_o \end{aligned}}$$

ODE
1^{st} order
nonlinear in T
nonhomogeneous
constant coeff.

How to solve?
↳ Numerically

SEMI INFINITE SOLID

$T(x,t) = ?$



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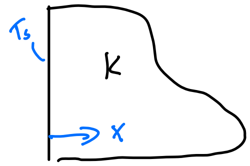

$x=0$
$t=0$
$T(x,t=0) = T_i$

$t>0$
$T(x=0,t) = T_s > T_i$

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \cancel{\frac{\dot{q}}{\rho c_p}}^{\,0}$$

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}}$$

PDE
2^{nd} order
linear
homogeneous
const. coeff

① IC
② BCs

SIMILARITY TRANSFORMATION

PDE
Hard to solve
$T(x,t)$

$\xrightarrow[\text{transformation}]{\text{similarity}}$

ODE
$T(\eta)$
$\eta = \eta(x,t)$
similarity variable
easier to solve

What is η?

α thermal diffusivity = diff coef. for heat

$$\alpha \sim \frac{L^2}{t} \Rightarrow L^2 \sim \alpha t \Rightarrow L \sim \sqrt{\alpha t}$$

"goes like"   time

the distance something diffuses goes like the square root of time.

$$\rightarrow \eta \sim \frac{x}{\sqrt{\alpha t}} \Rightarrow \boxed{\eta = \frac{x}{2\sqrt{\alpha t}}}$$

Dimensionless

add a constant to make it an =

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \qquad T = T(x,t)$$

ordinary derivative

$$\frac{\partial T}{\partial t} = ? = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t}$$

chain rule

partial derivative