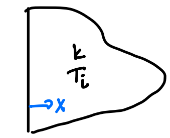
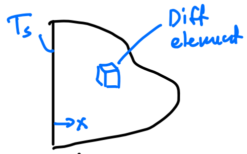
7A - 2ND CLASS



$x=0$
$t=0$

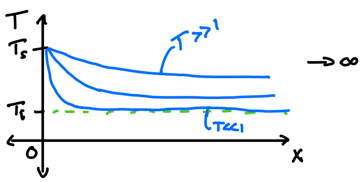


$t>0$
$T(0,t) = T_s > T_i$

$\rightarrow$ Heat Eqn $\rightarrow$ $\dfrac{\partial T}{\partial t} = \alpha \dfrac{\partial^2 T}{\partial x^2}$

Init Cond: $T(x, t=0) = T_i$

Boundy Cond:
1. $T(x=0, t) = T_s$
2. $T(x=\infty, t) = T_i$



Similarity Transformation $\quad \eta = \dfrac{x}{2\sqrt{\alpha t}}$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t} \qquad \eta = \frac{x}{2\sqrt{\alpha}} t^{-1/2}$$

$$= \frac{dT}{d\eta}\left(\frac{x}{2\sqrt{\alpha}}\left(-\frac{1}{2}\right)t^{-3/2}\right)$$

$$= \frac{dT}{d\eta}\left(\frac{-x}{2\sqrt{\alpha}}\,\frac{1}{2}\,t^{-1/2}\,t^{-1}\right) \Rightarrow \frac{dT}{d\eta}\left(\underbrace{\frac{-x}{2\sqrt{\alpha t}}}_{\eta}\,\frac{1}{2}t^{-1}\right)$$

$$= -\frac{dT}{d\eta}\frac{\eta}{2t}$$

$$\frac{\partial T}{\partial t} = \frac{-\eta}{2t}\frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \cdots = \frac{d^2 T}{d\eta^2}\frac{1}{4\alpha t}$$

So, $\dfrac{\partial T}{\partial t} = \alpha \dfrac{\partial^2 T}{\partial x^2} \longrightarrow \dfrac{-\eta}{2\cancel{t}}\dfrac{dT}{d\eta} = \cancel{\alpha}\dfrac{d^2T}{d\eta^2}\dfrac{1}{4\cancel{\alpha t}}$

So, $\boxed{\dfrac{d^2T}{d\eta^2} + 2\eta\dfrac{dT}{d\eta} = 0}$

ODE
linear
2nd order $\rightarrow$ 2 BC
homogeneous
variable coeff. (2η)

B.C. for η:

$(x=0, t) \rightarrow \eta = 0$
$(x=\infty, t) \rightarrow \eta = \infty$
$(x, t=0) \rightarrow \eta = \infty$
} only 2 conditions for η

$$\eta = \frac{x}{2\sqrt{\alpha t}}$$

So, $T(x=0,t) = T(\eta=0) = T_s$
$T(x=\infty, t) = T(\eta=\infty) = T_i$
$T(x, t=0) = T(\eta=\infty) = T_i$

So,

$$\frac{d^2T}{d\eta^2} + 2\eta\frac{dT}{d\eta} = 0$$

* similarity transform worked !

$$T(\eta=0) = T_s$$
$$T(\eta=\infty) = T_i$$

How to solve?

Seperate & Integrate,

dummy var

$$u \equiv \frac{dT}{d\eta}$$

Then,

$$\frac{du}{d\eta} + 2\eta u = 0$$

$$\int \frac{du}{u} + 2\int \eta \, d\eta$$

$$\ln u = \frac{-2}{2}\eta^2 + A_1$$

$$u = e^{-\eta^2 + A_1}$$

$$= e^{-\eta^2}\underbrace{e^{A_1}}_{A_2}$$

$$= A_2 e^{-\eta^2}$$

so,

$$\frac{dT}{d\eta} = A_2 e^{-\eta^2}$$

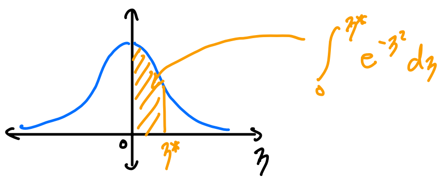
$$\int dT = A_2 \int_0^{\eta} e^{-\xi^2} d\xi$$

$$T(\eta) - \underbrace{T(0)}_{T_s} = A_2 \int_0^{\eta} e^{-\eta} d\eta$$

$$T(\eta) = T_s + A_2 \int_0^{\eta} e^{-\xi^2} d\xi$$

What is $e^{-\xi^2}$

Gaussian / Normal Distribution



This is represented as the "error" function : erf( )

$$\mathrm{erf}(\eta) \equiv \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\xi^2} d\xi$$